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Airport Situations and Games with Grey Uncertainty

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Abstract

Airport situations have much attention in recent years. Here, we focus on an appealing rule introduced by the economists Baker-Thompson called the Baker-Thompson rule which provides a fair and easy share for the costs of the landings. On the other hand, uncertainty is a daily presence in real life. It affects our decision-making and may have influence on cooperation. Recently, various economic and Operations Research situations under uncertainty are studied. In this paper, we deal with airport situations, where the costs of the pieces of the runway are given by grey numbers. In this context, we expand the Baker-Thompson rule as a solution concept. Some properties regarding an allocation problem of an airport situation under uncertainty is considered and grey solutions are proposed. We introduce grey Baker-Thompson rule. Further, we give the axiomatic characterization of the grey Baker-Thompson rule by using the major and the minor axioms where we give the first characterization by using $\mathcal{G}$IES, the second characterization is given by using $\mathcal{G}$CUR and the third characterization is given by using $\mathcal{G}$CLAST. Finally, we give an example to compare the grey solutions and grey Baker-Thompson rule.

1. Introduction

Mathematical models have been used to solve complex problems such as those in social sciences, economics, psychology, and politics. Game theory is a branch of applied mathematics that uses models to study interaction with formalized incentive structure game. This makes it easier to analyses all the game in the mathematical form or structure. By a game we mean not only recreational games like chess or poker. We also have in mind more serious games, such as contract negotiation between a labor union and a corporation, war negotiation or an election campaign.

Game theory can be defined as a mathematical framework consist of models and techniques that use to analyze the iterative decisions behavior of individuals concerned about their own benefit. These games are generally divided into two types, cooperative and competitive games.

Cooperative games is a game where all players are concerned about the overall benefits and they are not very worried about their own personal benefit. Thus, players fully cooperate with each other in order to achieve the highest possible overall benefit like football players in a team. Competitive games are games where every user is mainly concerned about his personal payoff and therefore all its decisions are made competitively and moreover selfishly. Thus, they are called non-cooperative games. Most of the two players' games are good example of this type.

Cooperative game theory has been enriched in the recent years with several models which provide decision making support in collaborative situations under uncertainty. These models are generalizations of the classical model regarding the type of coalition values. In classical cooperative game theory, the payoffs to coalitions of players are known with certainty, but when uncertainty is taken into consideration the characteristic functions are not real-valued as in the classical case. In our case, they capture the uncertainty on the outcome of cooperation in its different forms such as stochastic uncertainty, fuzzy uncertainty, interval uncertainty, ellipsoidal uncertainty.

To use the theory of grey numbers is reasonable in cases where uncertainty. Because it has been shown in the scientific literature that when predictions of zero probability events are concerned, no theoretical methods so far have been successful. According to recent publications, such a lack of success is mainly due to how information and consequent uncertainties are handled. In this study, we show how grey systems compare to other concepts of uncertain information [7].

In our daily lives uncertainty affects decision making in many situations. The problem airport situations under uncertainty is one of the situations that have attracted the attention of researchers in this field.

We consider the aircraft fee problem of the airport with one runway and suppose that the planes which are to land are classified into m kinds. In the classical airport situations for each $1 \leq j \leq m$, we denote the set of landings of planes of kind j by $N_j$and its carnality by n_j. Then, $N = \cup_{j=1}^{m} N_j$ represents the set of all landings. Let c_j represent the cost of a runway adequate for planes of kind j.

We assume that the kinds are ordered such that $0 < c_0 < c_1 < \ldots < c_m$. We consider the runway divided into m consecutive pieces P_j, $1 \leq j \leq m$, where P_1 is adequate for landings of planes of kind 1; P_1 and P_2 together for landings of planes of kind 2, and so on. The cost of piece P_j, $1 \leq j \leq m$, is the marginal cost $c_j - c_j - 1$. That is, every landing of planes of kind j contributes to the cost of the pieces P_k, $1 \leq k \leq j$, equally allocated among its users $\cup_{r=k}^{m} N_r$. Accordingly, [3] and [8] submitted the following rule:

$$BT_i = \sum_{k=1}^{j} \left[\sum_{r=k}^{m} n_r\right]^{-1} (c_k - c_{k-1}), \qquad (1)$$

whenever $i \in N_j$, which is known as Baker-Thompson rule. This rule provides a fair and direct cost for aircraft landing at airport.

Assume that the planes which are to land are classified into m types. For each $1 \leq j \leq m$, denote the set of landings of planes of type j by N_j and its cardinality by n_j. Then $N = \cup_{j=1}^{m} N_j$ represents the set of all landings. Consider the runway is divided into m consecutive pieces P_j,

$1 \leq j \leq m$, where P_1 is sufficient for landings of planes of type 1; P_1 and P_2 together for landings of planes of type 2, and so on.

In the sequel, we introduce the grey $w_j^{'}$ with nonnegative finite bounds represent the grey cost of piece P_j, $1 \leq j \leq m$. For a given airport grey situation $(N, (w_k^{'})_{k=1,\dots,m})$ the Baker-Thompson allocation for each player $i \in N_j$ is given by:

$$\mathcal{B}_i = \sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} w_k^{'} \in \left[\sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} \underline{w_k^{'}}, \sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} \overline{w_k^{'}}\right]. \quad (2)$$

Now, we recall some properties regarding an allocation problem of an interval airport situation. Formally, an allocation rule for an allocation problem is a map F associating each allocation problem$(N, (w_k)_{k=1,\dots,m}$, a unique point $F(N, (w_k)_{k=1,\dots,m}) \in \mathrm{IR}^N$ with $\sum_{i \in N} F_i(N, (w_k)_{k=1,\dots,m}) = [\sum_{k=1}^{j} \underline{w}_k, \sum_{k=1}^{j} \overline{w}_k]$.

An allocation rule F satisfies individual equal sharing (IIES) property if for every situation $(N, (w_k)_{k=1,\dots,m})$, $F_i(N, (w_k)_{k=1,\dots,m}) = \left[\frac{\sum_{k=1}^{j} \underline{w_k}}{\sum_{r=k}^{m} n_r}, \frac{\sum_{k=1}^{j} \overline{w_k}}{\sum_{r=k}^{m} n_r}\right] \geq \left[\frac{\sum_{k=1}^{j} \underline{w_k}}{n}, \frac{\sum_{k=1}^{j} \overline{w_k}}{n}\right]$ for each $i \in N_r$ and $r = 1, \dots, m$.

An allocation rule F satisfies collective usage right (ICUR) property if for every situation $(N, (w_k)_{k=1,\dots,m})$,

$$F_i(N, (w_k)_{k=1,\dots,m}) \leq \left[\sum_{k=1}^{j} \underline{w_k} (\sum_{l=1,\dots,r} n_l)^{-1}, \sum_{k=1}^{j} \overline{w_k} (\sum_{l=1,\dots,r} n_l)^{-1}\right],$$

for each $i \in N_r$ and $r = 1, \dots, m$.

An allocation rule F satisfies consistency on last group (ICLAST) property if for every situation $(N, (w_k)_{k=1,\dots,m})$ and for each $h \in N_m$

$$F_i(N, (w_k)_{k=1,\dots,m}) = F_i(\widehat{N}, (\widehat{w}_k)_{k=1,\dots,m}), i \in N \backslash \{h\}, \quad (3)$$

where $\widehat{N}_l = N_l$, $l = 1, \dots, m-1$, $\widehat{N}_m = N_m \backslash \{h\}$ and $\widehat{w}_l = w_l - F_h(N, (w_k)_{k=1,\dots,m})$, $l = 1, \dots, m$.

Baker-Thompson rule satisfies the properties above and do characterization by using them [6]. Then [1] extended these results to the interval setting to show that the Baker-Thompson rule satisfies the properties above. Our aim is to extend these results to the grey setting. In this paper, we give an axiomatic characterization of the grey Baker-Thompson rule. Our intuition is from [1] who study an axiomatic characterization of the interval Baker-Thompson rule.

The grey cost allocation rule $\mathcal{B}$ presented above called the grey Baker-Thompson rule. For the piece P_k of the runway the users are $\cup_{r=k}^{m} N_r$ meaning that there are $\cup_{r=k}^{m} n_r$ users. So, $(\sum_{r=k}^{m} n_r)^{-1} w_k^{'}$ is the equal cost share of each user of the piece P_k. This means that a player $i \in N_j$ contributes to the cost of the pieces $P_1, \dots, P_j$.

2. On the Grey Baker-Thompson Rule

In this section, we introduce the grey Baker-Thompson rule and give a characterization.

Theorem 2.1 Let $(N,(w_k^{'})_{k=1,\dots,m})$ be an airport grey situation. Then the grey Baker-Thompson rule $\mathcal{B}$ for each player $i \in N_j$ is $\mathcal{B}_i \in [\underline{\mathcal{B}_i}, \overline{\mathcal{B}_i}]$.

Proof. . By (2) we have for all $i \in N_j$

$$\mathcal{B}_i = \sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} w_k^{'} = \sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} [\underline{w_k^{'}}, \overline{w_k^{'}}]$$

$$\in \left[\sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} \underline{w_k^{'}}, \sum_{k=1}^{j} (\sum_{r=k}^{m} n_r)^{-1} \overline{w_k^{'}}\right] = [\underline{\mathcal{B}_i}, \overline{\mathcal{B}_i}] \quad (4)$$

Theorem 2.1 shows that one can calculate the lower bound of the grey Baker-Thompson rule by using the lower bounds of the grey costs and the upper bound of the grey Baker-Thompson rule by using the upper bounds of the grey costs.

We define an grey allocation rule for a given airport grey situation $(N,(w_k^{'})_{k=1,\dots,m})$ as a map F associating each allocation situation $(N,(w_k^{'})_{k=1,\dots,m})$ to a unique rule $F(N,(w_k^{'})_{k=1,\dots,m}) = F(N,([\underline{w_k^{'}}, \overline{w_k^{'}}])_{k=1,\dots,m}) \in \mathcal{G}(\mathbb{R})^N$ with

$$\sum_{i\in N} F_i(N,(w_k^{'})_{k=1,\dots,m}) = \sum_{i=1}^{m} [\underline{w_i^{'}}, \overline{w_i^{'}}] \in \left[\sum_{i=1}^{m} \underline{w_i^{'}}, \sum_{i=1}^{m} \overline{w_i^{'}}\right] \quad (5)$$

A grey allocation rule F satisfies grey individual equal sharing ($\mathcal{G}$IES) property if for every grey situation $(N,(w_k^{'})_{k=1,\dots,m})$, $(N,(\underline{w_k^{'}})_{k=1,\dots,m})$ and $(N,(\overline{w_k^{'}})_{k=1,\dots,m})$ satisfies IIES for each $i \in N_r$ and $r = 1,\dots,m$.

A grey allocation rule F satisfies grey collective usage right ($\mathcal{G}$CUR) property if for every grey situation $(N,(w_k^{'})_{k=1,\dots,m})$, $(N,(\underline{w_k^{'}})_{k=1,\dots,m})$ and $(N,(\overline{w_k^{'}})_{k=1,\dots,m})$ satisfies ICUR for each $i \in N_r$ and $r = 1,\dots,m$.

A grey allocation rule F satisfies grey individual consistency on last group ($\mathcal{G}$CLAST) property if for every grey situation $(N,(w_k^{'})_{k=1,\dots,m})$, $(N,(\underline{w_k^{'}})_{k=1,\dots,m})$ and $(N,(\overline{w_k^{'}})_{k=1,\dots,m})$ satisfies ICLAST for each $i \in N_r$ and $r = 1,\dots,m$.

Next we give some properties of the grey Baker-Thompson rule with the following proposition.

Proposition 2.1 The grey Baker-Thompson rule $\mathcal{B}$ satisfies $\mathcal{G}$IES, $\mathcal{G}$CUR, and $\mathcal{G}$CLAST.

Proof. The proof can be obtained by following the steps of [6] and [1] for $\underline{\mathcal{B}_i}$ and $\overline{\mathcal{B}_i}$ for each $i \in N_j$ and $j = 1,\dots,m$. Then, by using Theorem 2.1 we are done.

Now, we give an axiomatic characterization of the grey Baker-Thompson rule with the following theorem.

Theorem 2.2 The grey Baker-Thompson rule $\mathcal{B}$ is the unique rule satisfying $\mathcal{G}$IES, $\mathcal{G}$CUR and $\mathcal{G}$CLAST.

Proof. From Proposition 2.1 we know that $\mathcal{B}$ satisfies the three properties. We only need to show the uniqueness. For uniqueness, it is clear by [1] in which $\underline{\mathcal{B}_i}$ and $\overline{\mathcal{B}_i}$ for each $i \in N_j$ and $j = 1, \ldots, m$, are the unique allocations satisfying the three properties IIES, ICUR and ICLAST. Finally, by Theorem 2.1 we conclude that $\mathcal{B}_i \in [\underline{\mathcal{B}_i}, \overline{\mathcal{B}_i}]$ for each $i \in N_j$ and $j = 1, \ldots, m$ is unique. Hence $\mathcal{B}$ is the unique grey allocation satisfying $\mathcal{G}$IES, $\mathcal{G}$CUR and $\mathcal{G}$CLAST.

3. Grey numbers

The grey system theory initiated in 1982 by [4], is a new methodology that focuses on the study of problems involving small samples and incomplete information. As far as information is concerned, the systems which lack information, such as structure message, operation mechanism and behavior document, are referred to as Grey Systems. The grey system theory is one of the new mathematical theories born out of the concept of the grey set. It deals with uncertain systems with partially known information through generating, excavating, and extracting useful information from what is available. Uncertain systems with small samples and incomplete information exist commonly in the natural world. It is an effective method used to solve uncertainty problems with discrete data and incomplete information. In theory, random variables are regarded as grey numbers, and a stochastic process is referred to as a grey process. A grey system is defined as a system containing information presented as grey numbers; and a grey decision is defined as a decision made within a grey system.

Grey systems analysis consists mainly of grey incidence analysis, grey statistics, grey clustering, etc. Grey systems modeling is done mainly through generations of grey numbers or functions of series operators to find hidden patterns, if any. Then, the modeling is finished based on the concept of five-step-modeling. The concept of five-step-modeling consists of language model: network model, quantification of model, dynamical quantification of model and optimization of model [13].

We denote by $\mathcal{G}(\mathbb{R})$ the set of interval grey numbers in $\mathbb{R}$. Let $\otimes_1, \otimes_2 \in \mathcal{G}(\mathbb{R})$ with $\otimes_1 \in [\underline{a}, \overline{a}], \otimes_2 \in [\underline{b}, \overline{b}]$, $| \otimes_1 | = \underline{a} - \overline{a}$ and $\alpha \in \mathbb{R}_+$. Then

1. $\otimes_1 + \otimes_2 \in [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$;
2. $\alpha \otimes_1 = [\alpha \underline{a}, \alpha \overline{a}]$

By (1) and (2) we see that $\mathcal{G}(\mathbb{R})$ has a cone structure.

In general, the difference of $\otimes_1$ and $\otimes_2$ is defined by $\otimes_1 \ominus \otimes_2 = \otimes_1 + (-\otimes_2) \in [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$.

Different from the above subtraction we use a partial subtraction operator. We define $\otimes_1 \ominus \otimes_2$, only if $|\overline{\boldsymbol{a}} - \underline{\boldsymbol{a}}| \geq |\overline{\boldsymbol{b}} - \underline{\boldsymbol{b}}|$, by $\otimes_1 - \otimes_2 = [\underline{\boldsymbol{a}} - \underline{\boldsymbol{b}}, \overline{\boldsymbol{a}} - \overline{\boldsymbol{b}}]$ ([2]).

4. Grey solutions

We recall the definition of the grey solutions that is critical during this study ([9] and [12]). Now, we introduce some theoretical notions from the theory of cooperative grey games. For $w, w_1, w_2 \in IG^N$ and $w', w_1', w_2' \in \mathcal{G}G^N$ we say that $w_1' \in w_1 \leq w_2' \in w_2$ if $w_1'(S) \leq w_2(S)$, where $w_1'(S) \in w_1(S)$ and $w_2'(S) \in w_2(S)$, for each $S \in 2^N$. For $w_1', w_2' \in \mathcal{G}G^N$ and $\lambda \in \mathbb{R}_+$ we define $\langle N, w_1' + w_2' \rangle$ and $\langle N, \lambda w' \rangle$ by $(w_1' + w_2')(S) = w_1'(S) + w_2'(S)$ and $(\lambda w')(S) = \lambda w'(S)$ for each $S \in 2^N$. So, we conclude that $\mathcal{G}G^N$ endowed with "$\leq$" has a cone structure with respect to addition and multiplication with non-negative scalars above. For $w_1', w_2' \in \mathcal{G}G^N$ where $w_1' \in w_1, w_2' \in w_2$ with$|w_1(S)| \geq |w_2(S)|$ for each $S \in 2^N, \langle N, w_1' - w_2' \rangle$ is defined by $(w_1' - w_2')(S) = w_1'(S) - w_2'(S) \in w_1(S) - w_2(S)$.

a. Grey Shapley value

We call a game $\langle N, w' \rangle$ grey size monotonic if $\langle N, |w| \rangle$ is monotonic, i.e. $|w|(S) \leq |w|(T)$ for all $S, T \in 2^N$ with $S \subset T$. For further use we denote by $SM\mathcal{G}G^N$ the class of grey size monotonic games with player set N. The grey marginal operators and the grey Shapley value are defined on $SM\mathcal{G}G^N$. Denote by $\prod(N)$ the set of permutations $\sigma: N \to N$ of N. The grey marginal operator $m^\sigma: SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ corresponding to σ, associates with each $w' \in SM\mathcal{G}G^N$ the grey marginal vector $m^\alpha(w')$ of w' with respect to σ defined by

$$m_i^\sigma(w') := w'(P^\sigma(i) \cup \{i\}) - w'(P^\sigma(i)) \in [\underline{A_{P^\sigma(i)\cup\{i\}}} - \underline{A_{P^\sigma(i)}}, \overline{A_{P^\sigma(i)\cup\{i\}}} - \overline{A_{P^\sigma}(i)}],$$

for each $i \in N$, where $P^\sigma(i) = \{r \in N | \sigma^{-1}(r) < \sigma^{-1}(i)\}$, and $\sigma^{-1}(i)$ denotes the entrance number of player i. For grey size monotonic games $\langle N, w' \rangle$, $w'(T) - w'(S) \in w(T) - w(S)$ is defined for all $S, T \in 2^N$ with $S \subset T$ since $|w(T)| = |w|(T) \geq |w|(S) = |w(S)|$. We notice that for each $w' \in SM\mathcal{G}G^N$ the grey marginal vectors $m^\sigma(w')$ are defined for each $\sigma \in \prod(N)$, because the monotonicity of $|w|$ implies $\overline{A_{S\cup\{i\}}} - \underline{A_{S\cup\{i\}}} \geq \overline{A_S} - \underline{A_S}$, which can be rewritten as $\overline{A_{S\cup\{i\}}} - \overline{A_S} \geq \underline{A_{S\cup\{i\}}} - \underline{A_S}$. So, $w'(S \cup \{i\}) - w'(S) \in w(S \cup \{i\}) - w(S)$ is defined for each $S \subset N$ and $i \notin S$. Next, we notice that all the grey marginal vectors of a grey size monotonic game are efficient grey payoff vectors. The grey Shapley value $\Phi': SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ is defined by [9] as follows:

$$\Phi'(w') := \frac{1}{n!} \sum_{\sigma \in \Pi(N)} m^\sigma(w') \in \left[\frac{1}{n!} \sum_{\sigma \in \Pi(N)} m^\sigma(\underline{A}), \frac{1}{n!} \sum_{\sigma \in \Pi(N)} m^\sigma(\overline{A})\right], \tag{6}$$

for each $w' \in SM\mathcal{G}G^N$.

b. The grey Banzhaf value

The grey Banzhaf value $\beta: SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$, $\forall w' \in SM\mathcal{G}G^N$ is defined by

$$\beta(w') = \frac{1}{2^{|N|-1}} \sum_{i \in S} [w'(S) - w'(S \backslash \{i\})] \tag{7}$$

c. The $\mathcal{GCIS}$-value

The CIS-value [5] assigns to every player its individual worth, and distributes the remainder of the worth of the grand coalition *N* equally among all players [11].

The grey CIS-value assigns every player to its individual grey worth, and distributes the remainder of the grey worth of the grand coalition N equally among all players. The $\mathcal{GCIS}$-value $\mathcal{GCIS}: SM\mathcal{GG}^N \to \mathcal{G}(\mathbb{R})^N$ is defined by

$$\mathcal{GCIS}_i(w') = w'(\{i\}) + \frac{1}{|N|}\left[w'(N) - \sum_{j \in N} w'(\{j\})\right] \tag{8}$$

d. The $\mathcal{GENSC}$-value

The grey ENSC-value ($\mathcal{GENSC}$-value) assigns to every game w' the $\mathcal{GCIS}$-value of its dual game:

$$\mathcal{GENSC}_i(w') = \mathcal{GCIS}_i(w'^*) = \frac{1}{|N|}\left[w'(N) + \sum_{j \in N} w'(N\backslash\{j\})\right] - w'(N\backslash\{i\})) \tag{9}$$

The $\mathcal{GENSC}$-value assigns to every player in a game its grey marginal contribution to the "grand coalition" and distributes the remainder equally among the players.

e. The $\mathcal{GED}$-solution

The grey ED-solution ($\mathcal{GED}$-solution) $\mathcal{GED}: \mathcal{GG}^N \to \mathcal{G}(\mathbb{R})^N$ is given by

$$\mathcal{GED}_i(w') = \frac{w'(N)}{|N|}, \text{ for all } i \in N. \tag{10}$$

5. A Numerical Example

In this section, we calculate the grey solutions given in Section 4 and the Grey Baker-Thompson rule given in Section 2. We illustrate our results on Table 1.

Let $(N = \{1,2,3\}, (w'_k)_{k=1,2,3})$ be an airport grey situation with the grey cost $w'_1 \in [30,36]$, $w'_2 \in [40,50]$ and $w'_3 \in [100,120]$. Then, $w'(\phi) \in [0,0]$, $w'(1) \in [30,36]$, $w'(2) = w'(1,2) \in [70,86]$ and $w'(3) = w'(1,3) = w'(2,3) = w'(N) \in [170,206]$. The following table shows the interval marginal vectors of the game, where rows correspond to orderings of players and columns correspond to players.

Table 1: The grey Baker-Thompson rule and the grey solutions

Grey Solutions	Player 1	Player 2	Player 3
Grey Baker-Thompson rule	$\in [10,12]$	$\in [30,37]$	$\in [130,157]$
Grey Shapley value	$\in [10,12]$	$\in [30,37]$	$\in [130,157]$
Grey Banzhaf value	$\in [7.5,9]$	$\in [27.5,34]$	$\in [127.5,154]$
$\mathcal{GCIS}$-value	$\in [-3.\overline{3}, -4.\overline{6}]$	$\in [36.\overline{6}, 45.\overline{3}]$	$\in [136.\overline{6}, 165.\overline{3}]$
$\mathcal{GENSC}$-value	$\in [23.\overline{3}, 28.\overline{6}]$	$\in [23.\overline{3}, 28.\overline{6}]$	$\in [123.\overline{3}, 148.\overline{6}]$
$\mathcal{GED}$-value	$\in [56.\overline{6}, 68.\overline{6}]$	$\in [56.\overline{6}, 68.\overline{6}]$	$\in [56.\overline{6}, 68.\overline{6}]$

This application shows that Baker-Thompson rule can help us to provide a fair and easy share for the costs of the landings. We note that the grey Baker-Thompson rule is interesting at an ex-ante stage to inform users about what they can expect to pay for the landings of the runway. But, in other situations when all costs are known with certainty, the classical Baker-Thompson rule can be applied to pick up effective costs for each.

6. Conclusion and Outlook

Game theory, which has studied deeply the interaction between competing or cooperating individuals, plays a central role in these new developments.

Grey systems analysis consists mainly of grey incidence analysis, grey statistics, grey clustering, etc. Grey systems modeling is done mainly through generations of grey numbers or functions of series operators to find hidden patterns, if any.

Much of cooperative game theory is built around problem how to distribute the collective income in fair and rational manners. In a cooperative game, when the value of a subset of players is evaluated via a combinatorial optimization problem, subject to constraints of resources controlled by members in the subset, the input size is usually polynomial in the number of players.

To use the theory of grey numbers is reasonable in cases where uncertainty. Because it has been shown in the scientific literature that when predictions of zero probability events are concerned, no theoretical methods so far have been successful. According to recent publications, such a lack of success is mainly due to how information and consequent uncertainties are handled. In this study, we show how grey systems compare to other concepts of uncertain information.

In this study, we introduce an axiomatic characterization of the grey Shapley value on an additive cone of cooperative grey games. Inspiring was Shapley’s axiomatic characterization [10]. We notice that whereas the Shapley value is defined and axiomatically characterized for arbitrary cooperative TU games, the grey Shapley value is defined only for a subclass of cooperative grey games, called grey size monotonic games, and is axiomatically characterized only on the strict subset of grey size monotonic games. The restriction to the class of size monotonic games was imposed by the need to establish efficiency of interval marginal vectors, and consequently of the grey the Shapley value.

The problems of airports are best with uncertainties, in like this an informational environment is decision-making effects on operations cooperation and economy. We consider the aircraft fee problem of an airport with one runway [1]. This paper shows that cooperative grey game theory can help us to provide a fair and easy share for the costs of the landings by using Baker-Thompson rule.

For future work, the category of cooperative grey games are often applied to different economic and research Operational Research (OR) issues like bankruptcy situations, other airport situations and sequencing situations etc.

We note that the obtained results can be used in different application areas such as OR, economic and management situations etc.

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