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Allocation of Fixed Costs with Proportional Distribution and Random Data in Data Envelopment Analysis

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ABSTRACT

Data envelopment analysis is a method to evaluate the performance of a set of homogeneous decision-making units with multiple inputs and outputs. Allocation of fixed costs with data envelopment analysis has attracted the attention of many researchers. In classical data envelopment analysis, all inputs and outputs are precisely defined. However, this assumption is not always true in practical problems. One of the important methods to deal with uncertain data is to examine random data in DEA. In this article, we expand one of the fixed cost allocation models to the random mode using the proportional distribution method. Then we implement the model with a practical example.

1. Introduction

Common infrastructures are used in many organizations. The use of common infrastructure brings common costs. The main issue is how these types of expenses should be fair to different units Allocated. For example, we can refer to how to allocate costs for the joint production of general products [18-20], the division of the fixed cost of buying a common telecommunication cable [2], and the allocation of advertising expenses of a car manufacturer among its dealers [3, 16].

In a one-dimensional case, such as the profit that users get from shared infrastructure, the proportional distribution method has been considered the standard fixed cost allocation method [2,

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10, 21, and 23]. According to this method, each DMU has allocated a share of the fixed cost directly to the income from the common infrastructure. This method is widely used in practice to simplify calculations.

However, in many cases, fixed cost allocation [2] is not one-dimensional. Many researchers have tried to use data envelopment analysis (DEA) to solve such problems. DEA is an effective tool for evaluating the efficiency of decision-making units (DMU) based on multiple inputs and outputs. DEA has at least three advantages over other methods in solving fixed cost allocation problems. First, DEA can be used with multiple inputs and outputs for variables considered in allocation problems, which may not be one-dimensional in many cases. Second, units using a common infrastructure are commonly homogeneous and comparable. For example, they use the same inputs and produce similar outputs through the same production process [3], which shows that the requirement of homogeneity of DMUs [8] can be the conditions for using DEA in the cost allocation problem fulfil [16]. Thirdly, the DEA technique provides the possibility of investigating the impact of applicable allocation programs on performance evaluation based on the empirical characteristics of the production possibility set [16].

For the first time, Cook and Kress [3] did the cost allocation problem using DEA. In the Cook and Kress method [3], fixed cost is considered as an additional input criterion in performance evaluation. Based on two principles of efficiency reliability and parato-minimalism, the fair allocation was obtained by solving several linear programming problems. Considering the reliability of the Cook and Kress approach [3], Cook and Zhu [4] extended the Cook and Kress method from input-oriented to output-oriented and obtained a suitable (but not optimal and desirable) cost allocation. Lin [17] found that when some specific constraints are added to Cook and Zhou's method, there is probably no optimal solution, and improved their method by setting fixed objectives to achieve optimal cost allocation. Amirtimori and Korderstami proposed an alternative allocation method based on DEA by combining the efficiency reliability of Cook and Kress [3] and additional constraints of Beasley [2] to achieve a unique allocation. Beasley [2] presented a non-linear DEA-based cost allocation method, in which the effect of feasible and possible allocation plans on the average efficiency of all DMUs is considered and some additional constraints to obtain a unique cost allocation have been added. Li et al. [6] to this problem based on a hypothesis that there are some other costs, except the fixed cost, which are considered as input in the cost allocation problem. This issue has been addressed. They claimed that fixed costs should be combined with these costs to obtain a new input in the allocation and evaluation of efficiency. Based on this research, they proved that the DEA efficiency score of each DMU under evaluation is monotonically non-increasing to the cost assigned to it, and a unique allocation according to the minimization of the variance of the assigned costs. Presented in all DMUs. However, what has not been investigated in the literature is a clear explanation of the exact relationship between the traditional proportional distribution method and DEA technology in the fixed cost allocation problem. Also, what is the theoretical basis for using DEA technology in solving the fixed cost allocation problem?

Si et al [23], have investigated the basic concept of fairness in one-dimensional mode and extended the proportional distribution in the general mode of multi-input and multi-output. In this paper, we extend the model presented by Si et al. for the case where the data are random.

The concept of programming with stochastic constraints was introduced by Cooper et al. [17]. Ghasemi et al. [11], presented a new model using the concept of a common set of weights. Azadeh et al. have used the coverage analysis model with random data to calculate the efficiency of power companies [1]. Huang and Lee [9], Cooper et al. [5-7], and Ghasemi et al. [9], have also used the concept of programming with stochastic constraints. Khodabakshi et al. [13] have developed a collective model to calculate returns to scale for the case where the data is random. Khodabakhshi [14,15] has used this concept to develop the hyper-efficiency model like input and output. Mozaffari et al. [22], present two strategies for allocating fixed costs with undesirable data.

This article is organized as follows. Section 2 is dedicated to the background of data envelopment analysis. Section 3 introduces the proportional distribution method and its extension to cost allocation problems. Section 4 extends the model presented in Section 3 to the stochastic case. Section 5 is dedicated to the application of this method. Section 6 is the conclusion.

2. Background

Suppose $DMU_j\,(j=1,\dots,n)$ a set of homogeneous units with consumption m Entrance $x_{ij}\,(i=1,\dots,m)$,s output $y_{rj}\,(r=1,\dots,s)$ produces If DMU_d , the unit under evaluation, its efficiency is calculated using the CCR model, which is a fractional model as follows and was introduced by Charnes and his colleagues in 1978:

$$
\begin{aligned}
\theta^*_d = \text{Maximize} \quad & \frac{\sum_{r=1}^{s} u_r y_{rd}}{\sum_{i=1}^{m} v_i x_{id}} \\
\text{S.T}: \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j = 1,\dots,n \\
& u_r \ge 0 \qquad r = 1,\dots,s \\
& v_i \ge 0 \qquad i = 1,\dots,m
\end{aligned} \tag{1}
$$

u_r And v_i respectively, the weights of the outputs and inputs and θ_d^* Optimum efficiency for DMU_d Is.

In this model, the goal is to obtain optimal weights for inputs and outputs so that the efficiency of the d[th] unit is the highest possible value. According to the limitations of the model, the ratio of the sum of the weighted outputs to the sum of the weighted inputs of any of the units does not exceed one, so the maximum value that the objective function can take is one. On the other hand, according to the hypotheses of data envelopment analysis, all data are non-negative and there is at least one positive input and output among the data, and according to the maximum of the objective function, the lowest value that the objective function can take is greater than zero. Based on Chazens and Cooper transformations, the fractional model is transformed into the following linear model, which is known as the multiple CCR model.

$$
\begin{aligned}
\theta^*_d = \text{Maximize} \quad & \sum_{r=1}^{s} u_r y_{rd} \\
\text{S.T.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1,\dots,n \\
& \sum_{i=1}^{m} v_i x_{id} = 1 \\
& u_r \ge 0 \qquad r = 1,\dots,s \\
& v_i \ge 0 \qquad i = 1,\dots,m
\end{aligned} \tag{2}
$$

Definition 1. The d-unit is called CCR function or strong function if and only if $\theta_d^* = 1$ and at least one optimal weight set such as (U^*, V^*) exists so that the following relations are established:

$$\begin{cases} U^* = (u_1^*, \dots, u_s^*) \text{ , } (u_r^* > 0 \;\; r = 1, \dots, s) \\ V^* = (v_1^*, \dots, v_m^*) \text{, } (v_i^* > 0 \;\; i = 1, \dots, m) \end{cases} \tag{3}$$

Otherwise, the d-m unit of CCR is ineffective.

Proportional distribution method and allocation of fixed costs

It assumes that n common units distributed using a common fixed-cost infrastructure are positive R and the cost assigned to DMU_j also with R_j be shown so we have:

$$\sum_{j=1}^{n} R_j = R, R_j \geq 0, \forall j \tag{4}$$

This ensures that the allocated costs reach exactly R and the allocated cost amount R_j to each DMU_j, from zero to R.

One dimensional mode

Based on the proportional distribution method, each DMU allocates a share of the fixed cost directly to the profit it derives from the shared infrastructure. Suppose the profit of unit j in the use of common infrastructure, $y_j, j = 1, \dots, n$ then the proportional distribution criteria suggest that the cost allocated to each unit j should be as follows:

$$R_j = \frac{y_j}{\sum_{j=1}^{n} y_j} R, j = 1, \dots, n \tag{5}$$

In addition to simple calculations in allocation, formula (4) has at least two other advantages:

A)) To use a weight the same as $y_j, j = 1, \dots, n$ among all DMUs, it seems fair. To show this point more, formula (4) is changed to the following equation:

$$\begin{aligned} & R_j = u y_j, j = 1, \dots, n \\ & \sum_{j=1}^{n} R_j = R \\ & u, R_j \geq 0, \forall j \end{aligned} \tag{6}$$

In (6), all y_j, $j = 1,...,n$ are attached to the common weight u and indeed u uniquely in this case by the formula $u = \frac{R}{\sum_{j=1}^{n} y_j}$ Determined.

(b) There is an implicit assumption that if a unit's revenues from shared infrastructure increase, the cost allocated to that unit will also increase. The last limitation $u \geq 0$ in relation (6), this makes the implicit assumption effective.

2.3 Multi-input and multi-output mode

However, in many cases of fixed cost allocation [1-6], the fixed cost allocation problem is no longer a one-dimensional situation. Therefore, it is necessary to generalize the traditional proportional distribution from the one-dimensional state to the general multi-dimensional state. According to the two advantages of formula (4), a fair allocation in the multidimensional case must satisfy two principles:

(i) There is a common set of weights attached to variables in all DMUs. (ii) More income of a unit from common infrastructure increases the cost allocated to it. This shows that the use of common infrastructure has a positive effect on the income of the units. Conversely, the more inputs a DMU consumes, the lower the costs allocated to it. This implies the fact that the inputs and costs allocated in generating the revenues of each unit are interchangeable. Suppose unit j uses m inputs $x_{ij}\,(i = 1,...,m)$, s output $y_{rj}\,(r = 1,...,s)$ produces Based on two principles (i) and (ii), the allocated cost for unit j should be determined through a model as follows:

$$\begin{aligned} & R_j = \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \\ & \sum_{j=1}^{n} R_j = R \\ & u_r, v_i, R_j \geq 0 \end{aligned} \tag{7}$$

The relation (7) is the natural and normal generalization of the proportional distribution method in the general case of multiple inputs and multiple outputs. For ease of description, we call relation (7) a generalized proportional distribution equation for short. Suppose that the fixed cost is related to a shared infrastructure R and the cost is assigned to DMU_j With R_j It is shown that then the relative efficiency for each $DMU_d, d = 1,...,n$, can be calculated through the following model:

$$\begin{aligned}
& E^*_d = \text{Max} \frac{\sum_{r=1}^{s} u_r y_{rd}}{\sum_{i=1}^{m} v_i x_{id} + R_d} \\
& \text{S.T.} \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij} + R_j} \le 1 \qquad j = 1,...,n \\
& \sum_{j=1}^{n} R_j = R, \\
& u_r, v_i, R_j \ge 0
\end{aligned} \tag{8}$$

The first set of constraints in model (7) indicates that the fixed cost is considered as an independent input, whose weight is assumed to be one for convenience [4]. The second constraint of the model (7) guarantees that the total fixed costs are exactly R. E^*_d is the optimal value of the objective function for DMU_d Put in the model (7). By optimizing the relative efficiency in the model (7), each DMU can achieve a certain efficiency value, but it proposes a part of the cost allocation scheme. In general, the cost allocation scheme proposed by each DMU may be different from the other. The model presented by Si et al. is as follows:

$$\begin{aligned}
& Min \sum_{j=1}^{n} | R_j - \overline{R} | \\
& s.t. R_j = \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \\
& \sum_{j=1}^{n} R_j = R \\
& u_r, v_i, R_j \ge 0
\end{aligned} \tag{9}$$

Where in $\overline{R} = \frac{1}{n} \sum_{j=1}^{n} R_j = \frac{R}{n}$ and the goal is to ensure that the optimal allocation has the minimum gap and distance in the allocated costs among all DMUs. Model (8) is a non-linear programming. However, we change it linearly. For each j, put:

$$b_j = \frac{1}{2}\left(| R_j - \overline{R} | - R_j - \overline{R}\right), a_j = \frac{1}{2}\left(| R_j - \overline{R} | + R_j - \overline{R}\right)$$

Then we get the following relation:

$$|R_j - \overline{R}| = a_j + b_j, \forall j \qquad (10)$$
$$R_j = a_j - b_j$$

By substitution, a linear model is obtained as follows:

$$Min \sum_{j=1}^{n} \left(a_j + b_j\right) \qquad (11)$$
$$s.t. \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j = 0, j = 1,...,n$$
$$\sum_{j=1}^{n} \left(a_j - b_j\right) = R$$
$$a_j - b_j \geq 0, j = 1,...,n$$
$$a_j, b_j, u_r, v_i \geq 0, \forall r, i, j$$

If every j , i , And r , the optimal solution of the model $\left(u_r^*, v_i^*, a_j^*, b_j^*\right)$ is, then the optimal allocation of the base of each j is

$$R_j^* = a_j^* - b_j^*$$

1. Proportional distribution method and allocation of fixed costs in random mode

Assuming that $X_j = (x_{1j},...,x_{mj})^T$ and $Y_j = (y_{1j},...,y_{sj})^T$ Random input and output vectors respectively $DMU_j, j = 1,...,n$ to be Also vectors $X_j = (x_{1j},...,x_{mj})^T$ and $Y_j = (y_{1j},...,y_{sj})^T$ Also, the mathematical expectation vectors are the inputs and outputs of the decision-making unit, respectively. Suppose that the inputs and outputs have a normal distribution as follows:

$$x_{ij} \sim N\left(x_{ij}, \sigma_{ij}^2\right)$$

$$y_{rj} \sim N\left(y_{rj}, \sigma_{rj}^2\right)$$

Therefore, the random version of model 10 with possible restrictions is as follows:

$$\begin{aligned}
&Min\sum_{j=1}^{n}\left(a_j+b_j\right) \\
&s.t: \\
&P\left(\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-a_j+b_j\le 0\right)\ge 1-\alpha,\ j=1,\dots,n \\
&\sum_{j=1}^{n}\left(a_j-b_j\right)=R \\
&a_j-b_j\ge 0,\ j=1,\dots,n \\
&a_j,b_j,u_r,v_i\ge 0,\forall r,i,j
\end{aligned} \tag{12}$$

Where P is the concept of probability and error level α Specified by the administrator, it is a number between zero and one. Now we get the exact equivalent of model (12). Based on stochastic programming methods with probabilistic constraints, we obtain the first stochastic constraint of the model. For this purpose, consider the possible adverb of the unit under the jth evaluation:

$$P\left(\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-a_j+b_j\le 0\right)\ge 1-\alpha,\ j=1,\dots,n$$

By defining a non-negative covariate ε_j the above inequality becomes the following equality:

$$P\left(\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-a_j+b_j\le 0\right)=1-\alpha+\varepsilon_j,\ j=1,\dots,n$$

Tip: Suppose X is a random variable, and a, b and c are fixed numbers. If $P(X\le a)=c$ and $b\le a$ then there is $d\le c$ So that $P(X\le b)=d$

Using the tip above, non-negative variable s_j There is such that:

$$P\left(\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-a_j+b_j\le -s_j\right)=1-\alpha,\ j=1,\dots,n \tag{13}$$

We explain:

$$h_j=\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-a_j+b_j$$

Since every linear combination of normal random variables has a normal distribution, we have:

$$h_j \sim N\left(h_j, \sigma_j^2(u,v)\right)$$

$$h_j = E(h_j) = E\left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j\right) = \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j$$

$$\begin{aligned}
\sigma_j^2(u,v) &= Var\left(h_j\right) = Var\left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j\right) \\
&= Var\left(\sum_{r=1}^{s} u_r y_{rj}\right) + Var\left(\sum_{i=1}^{m} v_i x_{ij}\right) - 2\operatorname{cov}\left(\sum_{r=1}^{s} u_r y_{rj}, \sum_{i=1}^{m} v_i x_{ij}\right) \\
&= Var\left(\sum_{r=1}^{s} u_r y_{rj}\right) + Var\left(\sum_{i=1}^{m} v_i x_{ij}\right) - 2\sum_{r=1}^{s}\sum_{i=1}^{m} u_r v_i \operatorname{cov}\left(y_{rj}, x_{ij}\right) \\
&= \sum_{k=1}^{s}\sum_{r=1}^{s} u_r u_k \operatorname{cov}\left(y_{rj}, y_{kj}\right) + \sum_{k=1}^{m}\sum_{i=1}^{m} v_i v_k \operatorname{cov}\left(x_{ij}, x_{kj}\right) \\
&\quad - 2\sum_{r=1}^{s}\sum_{i=1}^{m} u_r v_i \operatorname{cov}\left(y_{rj}, x_{ij}\right)
\end{aligned}$$

Considering the random variable h_j the relation (13) is rewritten as follows:

$$P\left(h_j \le -s_j\right) = 1-\alpha, \; j = 1,\dots,n$$

$$P\left(\frac{h_j - h_j}{\sigma_j(u,v)} \le \frac{-h_j - s_j}{\sigma_j(u,v)}\right) = 1-\alpha, \; j = 1,\dots,n$$

By placing $Z_j = \dfrac{h_j - h_j}{\sigma_j(u,v)}$ and the knowledge to this point that Z_j has a standard normal distribution, we have:

$$P\left(Z_j \le \frac{-h_j - s_j}{\sigma_j(u,v)}\right) = 1-\alpha, \; j = 1,\dots,n$$

$$P\left(Z_j \le \frac{h_j + s_j}{\sigma_j(u,v)}\right) = \alpha, \; j = 1,\dots,n$$

$$\varphi\left(\frac{h_j + s_j}{\sigma_j(u,v)}\right) = \alpha \rightarrow \varphi^{-1}\left(\alpha\right) = \frac{h_j + s_j}{\sigma_j(u,v)}$$

Therefore, the definitive form of the first possible adverb will be as follows:

$$h_j + s_j - \sigma_j(u,v)\varphi^{-1}(\alpha) = 0$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j + s_j - \sigma_j(u,v)\varphi^{-1}(\alpha) = 0$$

Therefore, the definitive form of model (12) with possible restrictions is as follows:

$$\begin{aligned} & Min \sum_{j=1}^{n}\left(a_j + b_j\right) \\ & s.t: \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j + s_j - \sigma_j(u,v)\varphi^{-1}(\alpha) = 0, \; j = 1,\ldots,n \\ & \sum_{j=1}^{n}\left(a_j - b_j\right) = R \\ & a_j - b_j \geq 0, \; j = 1,\ldots,n \\ & a_j, b_j, u_r, v_i \geq 0, \forall r, i, j \end{aligned} \tag{14}$$

By defining non-negative variables λ_j Model (14) can be converted into a quadratic programming model as follows:

$$\begin{aligned} & Min \sum_{j=1}^{n}\left(a_j + b_j\right) \\ & s.t: \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - a_j + b_j + s_j - \lambda_j \varphi^{-1}(\alpha) = 0, \; j = 1,\ldots,n \\ & \sum_{j=1}^{n}\left(a_j - b_j\right) = R \\ & a_j - b_j \geq 0, \; j = 1,\ldots,n \\ & \lambda_j^2 = \sum_{k=1}^{s}\sum_{r=1}^{s} u_r u_k \operatorname{cov}\left(y_{rj}, y_{kj}\right) + \sum_{k=1}^{m}\sum_{i=1}^{m} v_i v_k \operatorname{cov}\left(x_{ij}, x_{kj}\right) \\ & -2\sum_{r=1}^{s}\sum_{i=1}^{m} u_r v_i \operatorname{cov}\left(y_{rj}, x_{ij}\right) \\ & a_j, b_j, u_r, v_i, \lambda_j \geq 0, \forall r, i, j \end{aligned}$$

3. Applied study

In this part, we show the application of the model with a numerical example. The data is related to 17 power companies with three inputs and two outputs. We acknowledge that the information in this section has been used from an article published by Azadeh et al. in 2015. The length of the network, the transfer capacity of the motor and the number of employees as inputs for the units under evaluation, the delivery of units and the service receiving area, are other parameters that have been taken as outputs for these units. Table 1 shows the average data of 17 power companies.

Table 1: Average input and output of 17 electricity companies

DMU	Network length	Transformer capacity	Number of Employees	Delivery of units	Service receiving area
DMU01	46639.12	5552.374545	2178.2727	673474.1	15761464
DMU02	58576.64	4532.874545	3194.4545	653070.27	7310036.1
DMU03	43194.89	4334.48	1925.0909	485824.99	10154340
DMU04	60704.45	14745.56273	7383.8182	2025151.6	28827171
DMU05	63148.09	4888.588182	2693.9091	567611.95	12032030
DMU06	20870.94	2000.537273	722.09091	228229.3	4653825.7
DMU07	8923.738	870.6290909	379.72727	84128.505	1993341.5
DMU08	21901.15	1449.53	1023.3636	152518.19	2496980.3
DMU09	32681.8	2642.637273	1175.9091	366351.99	4135314.2
DMU10	36803.91	2915.090909	1213.5455	266953.91	6263096.7
DMU11	2376.777	248.8627273	99.090909	103474.55	514711.64
DMU12	22705.96	1890.642727	1100.7273	409489.5	3503522.2
DMU13	38223.56	4215.131818	1522.8182	438400.39	6012477.5
DMU14	17843.09	2291	1107	113915.7	6025715.4
DMU15	13533.07	7117.841818	19726.909	130705.96	3348636.1
DMU16	53833	8117.878182	2982.4545	398050.68	16878761
DMU17	50858.91	6214.212727	2023.6364	527308.72	11033563

First, we calculate the covariance of inputs, covariance of outputs, and covariance of input-output using EXECL software. Table 2 shows the covariance of the entries. Table 3 is related to the covariance of the outputs. Table 4 shows the input-output covariance.

Table 2: Covariance of inputs

DMU		I1	I2	I3
DMU01	I1	32919138.89	6089896.594	-3297886.205
	I2	6089896.594	1138408.161	-563851.964
	I3	-3297886.205	-563851.964	935615.6529
DMU02	I1	20312747.69	3331339.389	-1284668.562
	I2	3331339.389	563626.841	-191155.3521

DMU		I1	I2	I3
	I3	-1284668.562	-191155.3521	395723.7025
	I1	14118986.68	2718022.525	-1656388.211
DMU03	I2	2718022.525	532739.7582	-328116.1782
	I3	-1656388.211	-328116.1782	510046.9917
	I1	57560524.61	19962580.67	-25223891.19
DMU04	I2	19962580.67	7150569.905	-8508970.537
	I3	-25223891.19	-8508970.537	56937874.88
	I1	51376776.26	5833255.289	-4142562.083
DMU05	I2	5833255.289	684979.1741	-382131.0093
	I3	-4142562.083	-382131.0093	975550.8099
	I1	3077412.571	894440.156	-230229.8564
DMU06	I2	894440.156	322718.9753	-85043.60521
	I3	-230229.8564	-85043.60521	58109.90083
	I1	921963.2237	138538.5534	-84778.49322
DMU07	I2	138538.5534	21747.27283	-11263.51025
	I3	-84778.49322	-11263.51025	14602.19835
	I1	43892824.54	1459545.037	-833995.5452
DMU08	I2	1459545.037	124930.4122	-74854.10364
	I3	-833995.5452	-74854.10364	165273.686
	I1	10612181.29	1928364.086	-923765.1438
DMU09	I2	1928364.086	495391.9857	-923765.1438
	I3	-923765.1438	-207961.1968	195491.3223
	I1	44786796.81	6027554.826	-2389055.496
DMU10	I2	6027554.826	833038.0826	-291253.1405
	I3	-2389055.496	-291253.1405	260651.5207
	I1	53299339.27	5370724.51	1169972.302
DMU11	I2	5370724.51	541937.4284	118754.5016
	I3	1169972.302	118754.5016	30432.44628
	I1	2626641.068	589490.9354	-366456.3917
DMU12	I2	589490.9354	133495.3555	-68009.7238
	I3	-366456.3917	-68009.7238	287023.2893
	I1	18801365.04	4442419.959	-1473482.99
DMU13	I2	4442419.959	1056662.367	-330370.6215
	I3	-1473482.99	-330370.6215	283864.876
	I1	4254857.719	1294375.091	-680000.5455
DMU14	I2	1294375.091	407534.1818	-174477.3636
	I3	-680000.5455	-174477.3636	295307.6364
	I1	5820831.459	257080.5484	71347030.42
DMU15	I2	257080.5484	346937890	-105966688.3
	I3	71347030.42	-105966688.3	3649789796
DMU16	I1	3674509538	-35411223.34	64630893.18
	I2	-35411223.34	2571152.508	-1420781.322

DMU		**I1**	**I2**	**I3**
	I3	64630893.18	-1420781.322	2820388.248
DMU17	I1	44236881.36	10391666	-3616673.579
	I2	10391666	2523718.653	-769534.0608
	I3	-3616673.579	-769534.0608	639552.7769

Table 3: Covariance of outputs

DMU		O1	O2
DMU01	O1	7.74997E+11	3.26811E+11
	O2	3.26811E+11	9.43273E+12
DMU02	O1	8.88052E+11	6.64891E+11
	O2	6.64891E+11	5.67323E+12
DMU03	O1	3.99454E+11	2.12673E+11
	O2	2.12673E+11	3.17985E+12
DMU04	O1	6.80582E+12	4.64883E+11
	O2	4.64883E+11	1.63011E+13
DMU05	O1	6.96038E+11	-4.61921E+11
	O2	-4.61921E+11	5.84443E+12
DMU06	O1	43314985180	43314985180
	O2	43314985180	1.34484E+12
DMU07	O1	11901803326	8527527242
	O2	8527527242	1.42557E+11
DMU08	O1	45736419394	32416632231
	O2	32416632231	5.67436E+11
DMU09	O1	2.04722E+11	25887779497
	O2	25887779497	7.21168E+11
DMU10	O1	1.10135E+11	72480210321
	O2	72480210321	1.90334E+12
DMU11	O1	97060508268	3.21817E+11
	O2	3.21817E+11	1.10319E+12
DMU12	O1	3.66648E+11	7.03119E+11
	O2	7.03119E+11	2.34206E+12
DMU13	O1	3.61325E+11	-9520874091
	O2	-9520874091	2.59555E+12
DMU14	O1	21525563980	-39255041811
	O2	-39255041811	2.31459E+12
DMU15	O1	27684102440	-35215356280
	O2	-35215356280	8.03819E+11
DMU16	O1	2.72741E+11	4.81007E+11
	O2	4.81007E+11	1.65328E+13
DMU17	O1	4.91644E+11	2.73354E+11
	O2	2.73354E+11	8.00125E+12

Table 5: Covariance of inputs and outputs

DMU		O1	O2
DMU01	I1	1001273433	17504815443
	I2	258124325	3213409691
	I3	146008366	-1870645854
DMU02	I1	212690831	2565106623
	I2	84826378.7	451000119
	I3	125132399	-902379839
DMU03	I1	425014420	6614734977
	I2	101617245	1292200666
	I3	75597389.2	-734081380
DMU04	I1	5027151116	29286890998
	I2	2308156292	9942012017
	I3	-3461064456	-1.1944E+10
DMU05	I1	-1478511747	17155156630
	I2	-104898141	1958051218
	I3	409574961	-1251595047
DMU06	I1	107419580	1980794158
	I2	6012385.31	577723237.8
	I3	12410786.2	-145831997
DMU07	I1	18024163.1	358136596.4
	I2	5097820.13	54883419.44
	I3	1596498.54	8527527242
DMU08	I1	-431440207	2785877833
	I2	15985803.7	258575795.2
	I3	14984085.4	-164413217
DMU09	I1	139390912	2741104551
	I2	-34747833.5	499022635.9
	I3	34497384.6	-239614585
DMU10	I1	225161893	8645078713
	I2	60648831.3	1183982237
	I3	46611761.2	-425952843
DMU11	I1	2273387658	2273387658
	I2	228893950	757289685.6
	I3	49797427.2	170061094.3
DMU12	I1	382253886	1938489402
	I2	99433039.3	459325353.7
	I3	113187519	-74311462.3
DMU13	I1	-243250118	4828549377
	I2	-27872365.1	1142102358
	I3	116316258	-753620157
DMU14	I1	35687210.2	1971103752
	I2	19276405.8	532167397

DMU		O1	O2
	I3	22209531.2	-351106068
	I1	6364531.29	1203039157
DMU15	I2	-756386090	1670139462
	I3	3161511051	-2.2661E+10
	I1	8418204854	-9.216E+10
DMU16	I2	282209802	6408771304
	I3	199413924	-3603912114
	I1	928241765	18652511804
DMU17	I2	350599993	4390106405
	I3	80712360.1	-1476240375

Suppose infrastructure renovation costs 1000 billion dollars. The amount of cost allocated to each unit per α according to Table 6, which was obtained using the GAMS software.

Amounts $\varphi^{-1}(\alpha)$, for values $\alpha = 0.5$, $\alpha = 0.4$, $\alpha = 0.3$, $\alpha = 0.2$, and $\alpha = 0.1$ Respectively 0, -0.25, -0.52, -0.84, and -1.28. For different values α Different fixed cost values are obtained for each DMU. Therefore, the amount of costs to the level of error α It Depends. Error level α, it shows the extent of not establishing the constraints of the problem and is determined by the manager before solving the problem. Therefore, a change in this level leads to different results. If $\alpha = 0.5$ then $\varphi^{-1}(\alpha) = 0$. Therefore, the results obtained in this case for the coverage analysis model of random data are similar to the results obtained from the coverage analysis model with deterministic data.

4. Conclusion

Proportional distribution is considered a standard fixed cost allocation method in the one-dimensional case. However it cannot deal with the problem of cost allocation in a multidimensional mode, and several researchers have used the DEA method to solve the problem in a multidimensional mode. In this article, we have developed the allocation of fixed costs with the proportional distribution method for the case where inputs and outputs are random. We have also shown the application of the method with an example.

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