

A Mathematical Model for Resilience Supply Chain Network Design in Uncertainty Approach

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ARTICLE INFO

Received: 2024/08/01

Revised: 2024/09/09

Accept: 2024/10/05

Keywords:

Resilience, Supply Chain,
Network Design,
Mathematical Model.

ABSTRACT

This research paper presents a comprehensive mathematical model for designing resilient supply chain networks under uncertainty. The model incorporates various factors that contribute to supply chain resilience, such as demand variability, disruptions, and supply chain complexity. By considering these elements, the model aims to optimize the network structure and decision-making processes to enhance its ability to withstand shocks and maintain operations during challenging times. The study begins with a thorough literature review that examines existing research on supply chain resilience and mathematical modeling techniques. Subsequently, a novel mathematical model is developed, incorporating key resilience metrics and decision variables. The model is then solved using advanced optimization algorithms to determine optimal network configurations. Numerical experiments are conducted to evaluate the performance of the proposed model. Different scenarios are simulated to assess the model's sensitivity to various uncertainties and its effectiveness in mitigating risks. The results demonstrate the model's capability to design resilient supply chains that can adapt to changing conditions and minimize disruptions. Finally, the paper concludes by summarizing the essential findings and contributions of the research. The limitations of the study are also discussed, along with potential avenues for future research.

1. Introduction

Supply chain networks (SCNs) play a vital role in the global economy, connecting suppliers, manufacturers, distributors, and retailers to ensure the efficient flow of goods and services. However, these networks are increasingly exposed to uncertainties, such as natural disasters,

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Available online 10/05/2024

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economic downturns, and geopolitical events, which can disrupt operations and lead to significant financial losses. To mitigate these risks and enhance their ability to withstand shocks, organizations are seeking to design and manage more resilient supply chains [1-3, 22]

Resilience in a supply chain refers to its capacity to absorb, adapt, and recover from disturbances. It involves building redundancy, flexibility, and agility into the network to minimize disruptions and maintain essential functions. Several factors contribute to supply chain resilience, including:

Demand variability: Fluctuations in demand can create imbalances between supply and demand, leading to stockouts or excess inventory.

Disruptions: Unexpected events, such as natural disasters, supplier failures, or transportation disruptions, can disrupt the flow of goods and services.

Supply chain complexity: SCNs with multiple tiers and geographically dispersed nodes can be more complex and, therefore, more vulnerable to disruptions (see Figure 1) [4-5, 23,24].



Figure 1: SCN Design (SCND).

To address these challenges, organizations must adopt a systematic approach to designing and managing resilient supply chains. Mathematical modeling can provide valuable insights into the complex relationships between different factors and help identify optimal network configurations [5-6, 25].

This research paper presents a novel mathematical model for resilience SCND under uncertainty. The model incorporates various factors that contribute to supply chain resilience, such as demand variability, disruptions, and supply chain complexity. Considering these elements, the model aims to optimize the network structure and decision-making processes to enhance its ability to withstand shocks and maintain operations during challenging times [7-8, 26,27].

This paper contributes to the existing body of knowledge by:

- Proposing a comprehensive Resilient SCND (RSCND) framework that integrates resilience in SCND.
- Developing a Robust Stochastic Optimization (RSO) model to optimize network design while considering renewable energy integration.
- Demonstrating the effectiveness of the proposed model through a case study in the industry

The remainder of this paper is organized as follows:

This research is arranged into five sections. Section 2 defines the literature review and recent studies in the area of RSCND and tries to show the gap in research. Section 3 suggests a methodology for calculation. Section 4 proposes the results of this research. Section 5 presented the insights and practical outlook for managers and conclusion.

2. Survey related works

The literature on supply chain resilience has grown significantly in recent years. It focuses on understanding the factors contributing to resilience and developing strategies for enhancing it. Several studies have explored the concept of resilience from different perspectives, including risk management, supply chain design, and operational decision-making.

One of the key themes in the literature is the importance of considering uncertainty in supply chain planning and decision-making. Various approaches have been proposed to incorporate uncertainty, such as scenario planning, robust optimization, and stochastic programming. These methods allow decision-makers to account for potential disruptions and develop more resilient supply chains [8-9, 28,29].

In addition to uncertainty, other factors that have been identified as critical to supply chain resilience include:

Redundancy: Having multiple suppliers or facilities can reduce the impact of disruptions.

Flexibility: The ability to adapt to changing conditions, such as shifts in demand or supply disruptions.

Agility: The capacity to respond quickly and effectively to unexpected events.

Collaboration: Strong relationships among supply chain partners can improve information sharing and coordination.

Mathematical modeling has been widely used to analyze and optimize SCNs. Various models have been proposed to address different objectives, such as cost minimization, lead time reduction, and risk mitigation. However, few studies have focused on designing resilient supply chains using mathematical modeling [9-10].

This literature review highlights the need for a comprehensive mathematical model that incorporates multiple factors contributing to supply chain resilience and can be used to optimize network design under uncertainty. The proposed model in this research aims to fill this gap by providing a framework for designing resilient supply chains that can withstand disruptions and maintain operations in challenging environments.

3. Problem Statement and Solution Approach

The proposed model is a mixed-integer linear programming (MILP) model. The decision variables include:

- **Location decisions:** Binary variables indicating whether a facility is opened at a specific location.
- **Allocation decisions:** Continuous variables representing the flow of goods between facilities.
- **Inventory decisions:** Continuous variables representing the inventory levels at different facilities.

The objective function is to minimize the total cost of the SCN, including fixed costs for opening facilities, variable costs for transportation and inventory, and disruption costs.

The constraints in the model include:

- **Demand satisfaction:** The total quantity of goods supplied to each customer must meet or exceed the demand.
- **Capacity constraints:** Each facility's production or storage capacity must not be exceeded.
- **Flow balance:** The total inflow of goods to a facility must equal the total outflow.
- **Inventory balance:** The inventory level at the beginning of a period plus the inflow minus the outflow must equal the inventory level at the end.
- **Disruption constraints:** These constraints model the potential impact of disruptions on the supply chain, such as reduced capacity or increased transportation costs [11-12].

Resilience Metrics

To incorporate resilience into the model, several metrics are considered:

- **Redundancy:** The number of alternative suppliers or facilities available for each product.
- **Flexibility:** The ability to adjust production or distribution plans in response to disruptions.
- **Agility:** The speed at which the supply chain can recover from disruptions.
- **Disruption risk:** The probability of disruptions occurring and their potential impact on the supply chain.

These metrics are incorporated into the objective function and constraints to ensure the optimized network resists disruptions.

Uncertainty Modeling

The model incorporates uncertainty through the use of scenarios. Different scenarios representing various potential disruptions and demand patterns are generated. The model is solved for each scenario, and the solutions are combined to obtain a robust solution less sensitive to uncertainty.

This section outlines the RSCND design methodology with renewable energy integration [12-15, 30].

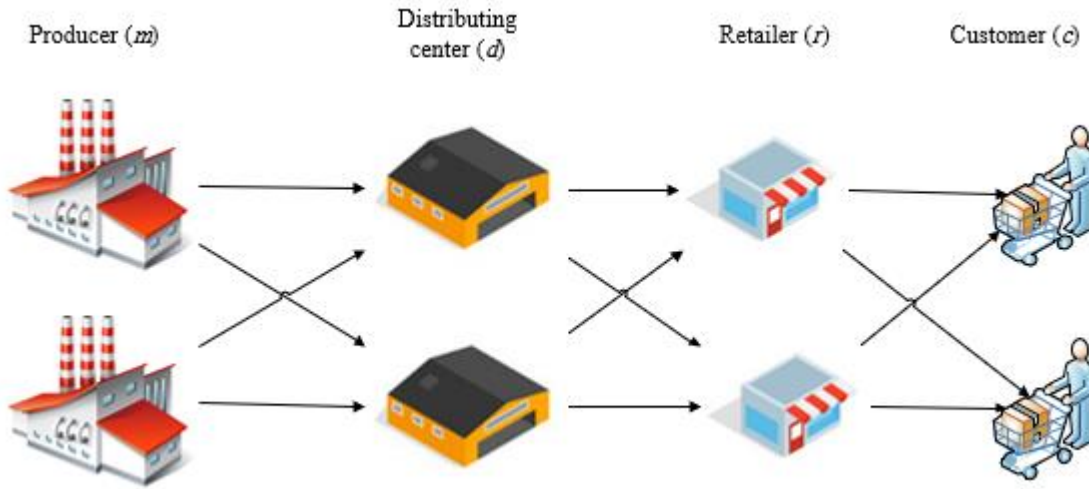


Figure 2: RSCND.

3.1. Mathematical model

The RSO model is a mathematical programming approach that optimizes the design of the RSCND network. The model considers various factors, including:

Based on the problem statement, these assumptions are assumed as follows:

Assumptions:

- Partial demand should be responded to, and the shortage is not permitted,
- Flow and capacity limitations with the resiliency approach are activated,
- A resilience strategy includes flexible capacity and redundancy in facility or multi-resource are set up (resiliency),
- Utilizing RSO is helpful for resilience in facing demand variation.

Sets, parameters, and variables definition:

Sets (Indices):

- m Set of producers (manufacturers), $m \in M = \{1, 2, \dots, \bar{m}\}$,
- d Set of distributor, $d \in D = \{1, 2, \dots, \bar{d}\}$,
- r Set of retailer, $r \in R = \{1, 2, \dots, \bar{r}\}$,
- c Set of customers, $c \in C = \{1, 2, \dots, \bar{c}\}$,

p Set of products (commodity), $p \in P = \{1, 2, \dots, \bar{p}\}$,

t Set of time period, $t \in T = \{1, 2, \dots, \bar{t}\}$,

s Set of scenarios, $s \in S = \{1, 2, \dots, s\}$.

Parameters	Description	Amount of parameter	Unit
de_{cpts}	Demand for product p in customer c in time t based on scenario s ,	U(3000,4000)	Number
Costs:			
fc_m	Set up cost for producer m ,	U(1,1.2)*1000000	Dollar
fd_d	Set up cost for disturbuter d ,	U(0.5,0.6)*1000000	Dollar
fr_r	Set up cost for retailer r ,	U(0.3,0.3)*1000000	Dollar
vmd_{mdpts}	Variable cost for transportation from producer m to disturbuter d for product p in time t based on scenario s ,	U(4,4.2)/1000	Dollar
vdr_{dpts}	Variable cost for transportation from disturbuter d to retailer r for product p in time t based on scenario s ,	U(3.9,4)/1000	Dollar
vrc_{rcpts}	Variable cost for transportation from retailer r to customer c for product p in time t based on scenario s ,	U(3,4)/1000	Dollar
Capacity:			
Cpm_{mpts}	Capacity of producer m for product p in time t based on scenario s ,	U(40500,41000)	Number
Cpd_{dpts}	Capacity of disturbuter d for product p in time t based on scenario s ,	U(38500,39000)	Number
Cpr_{rpts}	Capacity of retailer r for product p in time t based on scenario s ,	U(45000,46000)	Number
Other parameters			
p_s	Scenario probability s ,	1/ S	Percent
prm_m	Access level of producer m ,	U(95,98)	Percent
prd_d	Access level of disturbuter d ,	U(95,98)	Percent

pr_r	Access level of retailer r ,	U(95,98)	Percent
δ	Resiliency coefficient.	80	Percent

Decision variables:

Binary (zero-one) variables:

- xm_m Equal one, if producer m is set up; else zero,
- xd_d Equal one, if distributor d is set up; else zero,
- xr_r Equal one, if retailer r is set up; else zero,

Positive (Continues) variables:

- ymd_{mdpts} Flow quantity from producer m to distributor d for product p in time t based on scenario s ,
- ydr_{drpts} Flow quantity from distributor d to retailer r for product p in time t based on scenario s ,
- yrc_{rcpts} Flow quantity from retailer r to customer c for product p in time t based on scenario s ,

Auxiliary (slack) variables:

- FC Total fixed cost,
- VC_s Total variable cost for scenario s ,
- Γ_s Total fixed and variable cost for scenario s ,

Model 1: RSCND.

$$\text{minimize } Z = \sum_s p_s \Gamma_s, \tag{1}$$

subject to:

Cost constraints:

$$\Gamma_s = FC + VC_s, \tag{2}$$

$$FC = \sum_m fm_m xm_m + \sum_d fd_d xd_d + \sum_r fr_r xr_r, \tag{3}$$

$$VC_s = \sum_p \sum_t \left(\sum_m \sum_d ymd_{mdpts} ymd_{mdpts} + \sum_d \sum_r ydr_{drpts} ydr_{drpts} + \sum_r \sum_c yrc_{rcpts} yrc_{rcpts} \right), \quad \forall s \tag{4}$$

Balance requirements (Forward flow):

$$\sum_r yrc_{rcpts} \geq \delta de_{cpts}, \quad \forall c, p, t, s \tag{5}$$

$$\sum_d ydr_{dpts} \geq \sum_c yrc_{rcpts}, \quad \forall r, p, t, s \quad (6)$$

$$\sum_m ymd_{mdpts} \geq \sum_r ydr_{dpts}, \quad \forall d, p, t, s \quad (7)$$

Resiliency strategy (flexible capacity):

$$\sum_c yrc_{rcpts} \leq prr_r Cpr_{rpts} xr_r, \quad \forall r, p, t, s \quad (8)$$

$$\sum_r ydr_{dpts} \leq prd_d Cpd_{dpts} xd_d, \quad \forall d, p, t, s \quad (9)$$

$$\sum_d ymd_{mdpts} \leq pm_m Cpm_{mpts} xm_m, \quad \forall m, p, t, s \quad (10)$$

$$\min \left\{ \frac{\sum_m xm_m}{|M|}, \frac{\sum_d xd_d}{|D|}, \frac{\sum_r xr_r}{|R|} \right\} \geq \delta \quad (11)$$

Decision variables:

$$xm_m, xd_d, xr_r \in \{0,1\}, \quad \forall m, d, r \quad (12)$$

$$ymd_{mdpts}, ydr_{dpts}, yrc_{rcpts} \geq 0, \quad \forall m, d, r, c, p, t, s \quad (13)$$

The objective function (1) minimizes the cost function for all scenarios. Constraint (2) presents fixed and variable costs for the facility and each scenario. Constraint (3) shows the fixed cost for the facility. Constraints (4) show variable costs for setting up facilities for each scenario. Constraints (5) - (7) present forward flow quantity constraints, including demand satisfaction and balance between forward flow facilities. Constraints (8) to (10) state capacity constraint with a flexible approach as a resiliency strategy dependent on the scenario. Constraints (11) explain redundancy and multi-source constraint as a second resiliency strategy greater than the resiliency coefficient. Constraints (12) define activation binary variables for locations and the pillar of SCND that is set up if equal to one. Constraints (13) define the flow quantity variables that is positive or non-negative variables between the forward and reverse of CLSC.

The objective function of the RSO model aims to minimize the expected total cost of the SCN across all demand scenarios. This includes production and transportation costs. The model also incorporates constraints related to capacity limitations, demand satisfaction, and material flow balance.

3.2. Solution Approach

The RSO model is a complex mixed-integer linear program (MILP) that can be solved using specialized optimization software. The solution process involves:

1. Formulating the mathematical model with sets, parameters, and decision variables.
2. Defining the objective function and constraints.
3. Specifying the demand scenarios and their associated probabilities.
4. Utilizing optimization software to solve the model and obtain the optimal network design (see Figure 3) [15-21, 31].

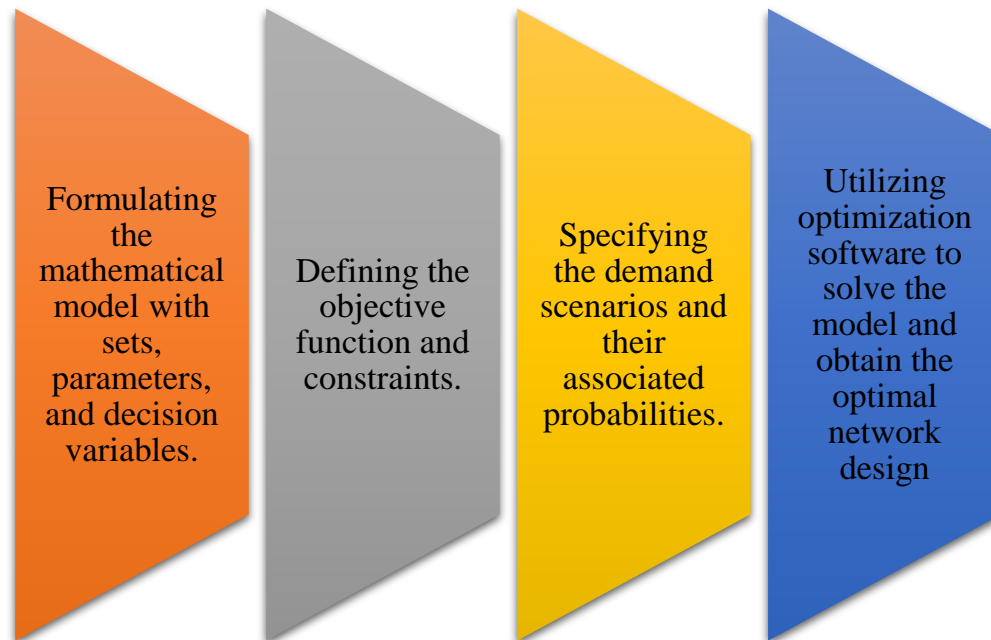


Figure 3: Solution approach.

4. Results and discussion

This section presents a case study to demonstrate the effectiveness of the proposed RSCND framework and RSO model. The case study considers an automation part supply chain with a network of potential manufacturing facilities, distributors, retailers, and customer markets.

Data on demand, production costs, transportation costs, and facility capacities are collected for each element in the network. Additionally, historical data or expert judgment is used to estimate disruption probabilities for different scenarios (e.g., natural disasters and economic downturns).

Multiple demand scenarios are created to represent potential disruptions. These scenarios may involve fluctuations in demand at specific customer markets or disruptions affecting particular facilities. The probability of each scenario occurring is also defined.

The RSO model is implemented in a mathematical programming software package like CPLEX. The collected data and defined scenarios populate the model parameters. The model is then solved to obtain the optimal design of the RSCND network.

The solution from the RSO model provides insights into the optimal configuration of the SCN. The specific results will depend on the input data and chosen scenarios. However, the case study demonstrates the effectiveness of the RSCND framework and RSO model in designing a resilient SCN that minimizes environmental impact.

Table 1. A number of indices and the cost function of the case study.

Problem	$ M D R C P T S $	Cost (Dollar)	Time (second)
Main model	3.3.3.3.3.3.3	3796535.007	0.277

Table 2. Final locations for RSCND.

Variables	City		
Manufacturer (xm_m)	Tabriz	Tehran	Mashhad
	0	1	1
Distributor (xd_d)	Sanandaj	Tehran	Mashhad
	0	1	1
Retailer (xr_r)	Sanandaj	Tehran	Mashhad
	0	1	1



Figure 4: Facility components.



Figure 5: Results of RSCND.

4.1. Analysis of resiliency coefficient

In this section, the resiliency coefficient (Ω) is changed between 10% to 60%. As can be seen, varying the resiliency coefficient increases cost function (see Table 3, Figure 6, and Figure 7). It is considered that when the resiliency coefficient increases, the mathematical model wants to increase responsibility. As a result, the cost function increases.

Table 3. Number of indices and cost function of the case study.

Problem	Resiliency coefficient (Ω)	Cost (Dollar)	Time (second)
Main model	10%	1857246.735	0.265
	20%	1857246.735	0.289
	30%	1857246.735	0.269
	40%	3796535.007	0.237
	60%	3796535.007	0.277

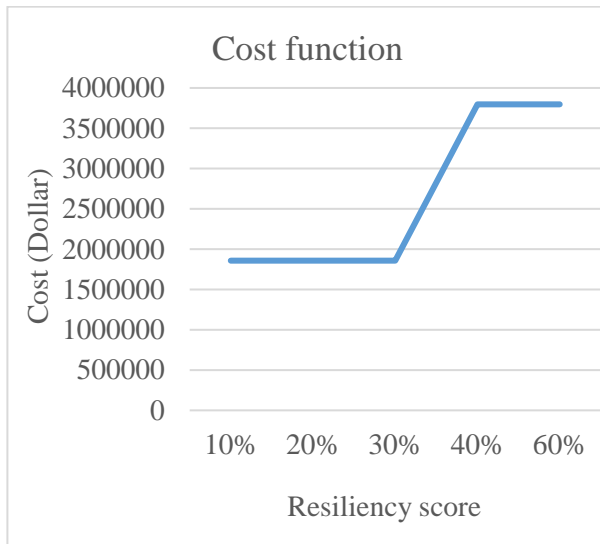


Figure 6: Analysis of resiliency coefficient on the cost function.

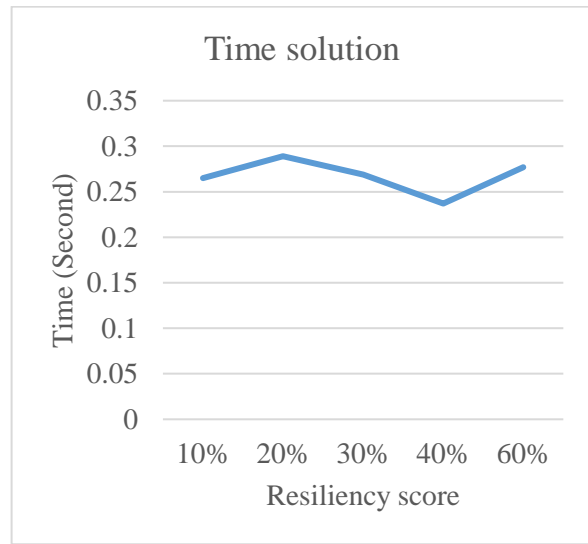


Figure 7: Analysis of resiliency coefficient on time solution.

5. Conclusion

This research paper presents a comprehensive mathematical model for RSCND under uncertainty. The model incorporates various factors contributing to supply chain resilience, such as demand variability, disruptions, and complexity. By considering these elements, the model aims to

optimize the network structure and decision-making processes to enhance its ability to withstand shocks and maintain operations during challenging times.

The numerical experiments conducted in this study demonstrate the effectiveness of the proposed model in designing resilient supply chains. The model can identify optimal network configurations that balance cost and resilience, ensuring the supply chain is not overly vulnerable to risks.

The findings of this research contribute to the growing body of knowledge on supply chain resilience and provide a valuable tool for organizations seeking to enhance their resilience. Future research could extend the model to incorporate additional factors, such as sustainability and social responsibility, and explore alternative optimization approaches.

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