



Impact of Image Processing on Process Improvement in a Manufacturing Industry

Sina Seifi^a, Rassoul Noorossana^b

^a Industrial Engineering Department, Iran University of Science and Technology, Tehran, Tehran.

^b Information Systems and Operations Management Department, College of Business, University of Central Oklahoma, Edmond, OK, 73034, United States.

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ABSTRACT

Image data has a significant impact on streamlining production processes by representing visual information, thereby improving overall organizational efficiency. Extracting valuable insights from image data is crucial for monitoring and enhancing statistical processes in manufacturing industries. This study introduces a new method based on image processing and fuzzy transform approach to process improvement in the manufacturing industry. The information is monitored using an exponentially weighted moving average chart (EWMA) control chart to detect change points. Furthermore, a real case study in the tile manufacturing industry, along with various numerical examples, is examined under different scenarios to assess the effectiveness of image processing on process improvement with the proposed method. The results of experimental tests show promising performance for the proposed approach.

1. Introduction

Image processing has the potential to enhance quality control in manufacturing by facilitating quicker, more precise, and more consistent inspection of products and components. This could lead to lower labor expenses and a decrease in the frequency of human mistakes. Some typical uses of image processing in quality control include identifying defects, measuring dimensions, and recognizing patterns. Defect detection entails identifying and pinpointing scratches, cracks, dents, or stains on the surface or inside of a product. Dimensional measurement involves assessing the measurements of a product or component, such as length, width, height, diameter, or angle. Pattern

^a Corresponding author email address: rnoorossana@uco.edu (Rassoul. Noorossana).

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recognition is utilized to identify and categorize patterns, such as logos, labels, barcodes, or characters on a product or package. All these applications can be employed to verify product information like batch number, expiration date, or product code.

Monitoring processes with statistical methods, known as Statistical Process Monitoring (SPM), is essential for upholding quality control in both manufacturing and service sectors. The method helps businesses improve processes and better meet customer demands. Control charts are essential for identifying different causes of variation, distinguishing between 'assignable' (non-random) and 'common' (random) causes. Common causes, which are inherent in production, typically have a smaller impact compared to assignable causes. Therefore, by analyzing and reducing process variations, companies can ultimately outperform their competitors as quality is inversely proportional to variation. Recently, there has been growing interest among researchers in uncovering unknown correlations and hidden patterns within diverse data sets to enhance production processes and meet customer requirements effectively. Advancements in machine vision technology and computer science have enabled the efficient collection of various data types, such as products and medical images or videos, at a reasonable cost. However, selecting appropriate statistical methods to monitor such large data sets poses a significant challenge in terms of quality control. Researchers have extensively explored image compression techniques and their various applications in image processing.

Roman-Gonzalez [1] conducted extensive investigations into different applications of these techniques. Similarly, Liu et al. [2] presented and statistically compared several image fusion algorithms. Duchesne et al. [3] offered a thorough framework and summary of techniques, as well as their applications, in multivariate image processing. Also, Singh and Manchanda [4] introduced a novel image compression method based on wavelets.

A new method to image compression and reconstruction, image fusion, and time-series processing introduced by Perfilieva and Hodakova [5], the methodology has since seen extensive development and application. Perfilieva [6] established the fundamental principles of image reduction and illustrated its real-world uses through practical applications. Building on this groundwork, Perfilieva [7] explored advanced applications of image reduction in different scopes. Further advancing the field, Perfilieva and Vlašánek [8] proposed a novel technique for image reconstruction, utilizing the approximation properties of the image reduction to remove noise effectively.

Expanding on this technology, Min et al. [9] introduced a new method that leverages components of the image for industrial applications. In another innovative study, Perfilieva and Adamczyk [10] presented a novel method that combines the image reduction approach with Laplacian values to analyze key points in an image. This method signifies a step forward in enhancing the analytical capabilities of image processing tools. Molek and Perfilieva [11] proposed a new technique that integrates convolutional neural networks (CNN) with reduced image components to categorize different images in image processing applications. This hybrid approach aims to harness the strengths of both CNN and reduced image for improved image classification. Furthering the application of the image reduction in manufacturing industry, Linh and Diep [12] employed this method in conjunction with swarm and evolutionary algorithms for image compression.

In recent years, researchers have combined image processing with Statistical Process Control (SPC) for process monitoring. Megahed et al. [13] improved the monitoring of image data by using the Generalized Likelihood Ratio (GLR) control chart, showing how traditional SPC tools can be applied in new contexts. Following this, Koosha et al. [14] introduced a new wavelet-based method for Statistical Process Monitoring (SPM). They applied wavelet transformation to extract features from image data, which were then monitored using a control chart across different time intervals to assess process performance. Adding to this area of study, Colosimo [15] proposed an innovative approach to modeling and monitoring, aiming to achieve zero-defect manufacturing. This method stands out for its integration of new inspection solutions and rapid multi-stream sensors capable of handling spatial and image data, enhancing the ability to effectively monitor complex manufacturing processes. Ledari [16] proposed a new approach based on fuzzy method for supply chains by considering disruption risk and special order. Ali [17] proposed a new method to design and develop a multipurpose shoeshine chair. He used quality function deployment, concept generation, concept selection, and economic analysis for design and development of the product.

The research gaps identified through these studies are summarized in Table 1, providing a structured overview of the potential areas for future development in the field of image-based process monitoring.

The research gaps identified through these studies are summarized in Table 1, providing a structured overview of the potential areas for future development in the field of image-based process monitoring.

Table 1. Comparative analysis of research gaps in the literature

References	Image data	Image compression	Image reconstruction	Statistical Quality control	Control chart	Manufacturing industry
Roman-Gonzalez [1]	✓	✓	-	-	-	-
Liu et al [2]	✓	-	-	-	-	✓
Duchesne et al [3]	✓	-	-	-	-	-
Singh and Manchanda [4]	✓	✓	-	-	-	✓
Perfilieva and Hod'áková [5]	-	-	-	-	-	-
Perfilieva [6]	-	-	-	-	-	-
Perfilieva [7]	-	-	-	-	-	-
Perfilieva and Vlašánek [8]	✓	-	✓	-	-	-
Min et al. [9]	-	-	-	-	-	-
Perfilieva and Adamczyk [10]	✓	-	-	-	-	-
Molek and Perfilieva [11]	✓	-	-	-	-	-
Linh and Diep [12]	✓	✓	-	-	-	-
Megahed et al [13]	✓	✓	-	✓	✓	-
Koosha et al [14]	✓	✓	-	✓	✓	✓
Colosimo [15]	✓	-	-	-	✓	-
Ledari [16]	-	-	-	-	-	✓
Ali [17]	-	-	-	✓	-	✓
Proposed Method	✓	✓	-	✓	✓	✓

Based on previous studies, our paper investigates the impact of image processing on process improvement based on monitoring image data. We have developed a technique that significantly decreases the number of bits needed to encode an image, resulting in considerable compression while maintaining image quality. Furthermore, we use the exponentially weighted moving average chart (EWMA) control chart to oversee the image components of the manipulated image data.

In the rest of this paper, the content is organized as follows. Section 2 explores the basic principles, addressing the issues related to image reduction. Section 3 presents our new approach, which integrates an innovative image reduction technique with image data monitoring methods based on the control chart. The assessment of this new approach, illustrated with a real case study, is described in Section 4. Lastly, Section 5 displays the outcomes obtained from the application of our methodology.

2. Image reduction

Image compression involves reducing the size of a graphics file without significantly impacting the image quality. This allows for more efficient use of storage space and faster transmission over the internet. In other words, Image compression reduces the size of image files, typically by removing bytes or using compression algorithms to rewrite the file. This process is essential for optimizing images to ensure fast loading on websites and applications.

A grayscale image consists of varying shades from black to white, making it easier to analyze compared to other types of images. A grayscale image can be identified with a single intensity function as $u : [1, N] \times [1, M] \rightarrow [0, 255]$, where the domain $[1, N] \times [1, M] = \{(i, j) | i = 1, \dots, N; j = 1, \dots, M\}$ and the range $[0, 255]$ contain a natural number. Furthermore, a gray-scale reduced image is described as $\bar{u} : n \times m \rightarrow [0, 255]$ of u which is determined by the reduction ratio $\rho = \frac{NM}{nm}$ (that is generally written as $\rho : 1$). Considering u and \bar{u} with different domains, compressed image \bar{u} transformed into the enlarged image \hat{u} (to compare this image), with a domain $[1, N] \times [1, M]$ by reconstruction procedure.

Quality control is crucial for any company or organization that produces a product or provides a service. Quality control helps to improve customer satisfaction by consistently delivering high-quality products or services, reducing resource wastage, and increasing productivity and profit for the company. Manufacturing a product is costly and time-consuming, and without control measures, it can be risky. If a manufacturer sends defective and low-quality products to its customers, it will be responsible for any injuries or problems caused by the use of that product. Quality means suitability for the intended purpose or the degree of compliance or non-compliance of the outputs of a process or the process itself with the specified requirements and principles. Quality control (QC) is the process by which businesses ensure that the quality of their products

is maintained or improved. Quality control involves testing units of product and determining whether they conform to final product specifications. The purpose of this testing is to identify any need for corrective actions in the production process. Effective quality control helps organizations to meet the demands of their customers and provide them with superior products. Quality control inspectors, following the principles of quality control, ensure that defective or unsafe products are identified and their causes are investigated.

In order to assess the impact of image processing on enhancing statistical processes, we begin by compressing the image and reducing its dimensions based on fuzzy transform approach. This step helps to expedite the analysis of image data. Next, the compressed image is subjected to analysis using a control chart to evaluate the effectiveness of the proposed approach. If the pixels of the compressed image are defined as follows F_1, \dots, F_n , considering an image function as $u: [1, N] \times [1, M] \rightarrow [0, 255]$, the components of the compressed image based on the fuzzy transform approach can be calculated based on following formula (Perfilieva et al., [18]):

$$U_{kl} = \frac{\sum_{i=1}^N \sum_{j=1}^M u(i, j) A_k(i) B_l(j)}{\sum_{i=1}^N \sum_{j=1}^M A_k(i) B_l(j)} \quad (1)$$

where U_{kl} , $k = 1, 2, \dots, n$, $l = 1, 2, \dots, m$, are components of compressed images which can be analyzed based on the control chart. Firstly, we create a 1D vector based on the components of a compressed image. Considering that there are different techniques for transforming a 2D matrix to a 1D vector, in this paper, we used a linear approach for constructing a set of 1D signals. Therefore, the vector of the compressed image of U_{kl} is as follows:

$$V = [\beta_{11}, \beta_{12}, \dots, \beta_{1m}, \beta_{21}, \beta_{22}, \dots, \beta_{2m}, \dots, \beta_{n1}, \beta_{n2}, \dots, \beta_{nm}] \quad (2)$$

where m is the number of components which is calculated based on a ρ ratio which determines the number of pixels specified for each F-transform component. Parameter β_{kl} describes as the l^{th} component for the k^{th} row of the reduced image intensity matrix. Each component is crucial in image processing. If a fault happens in any part of the image after applying the compression method, the proposed control chart will be altered, and this alteration can be identified using the suggested control chart.

3. Process monitoring based on EWMA

Different control charts have been created for monitoring statistical processes when data are represented as images. The EWMA control chart has been widely used in statistical process monitoring to distinguish between in-control and out-of-control states of a process.

The EWMA chart, also known as the exponentially weighted moving average chart, is a control chart used to monitor data from a business or industrial process. Unlike other control charts that treat individual samples separately, the EWMA chart tracks the weighted moving average of all previous sample means. It assigns decreasing weights to samples, with recent samples having the highest weight and distant samples contributing less. While the chart is based on the normal distribution, it is also robust when dealing with non-normally distributed quality characteristics. Additionally, there is an adapted version of the chart for quality characteristics modeled by the Poisson distribution. It's important to note that the chart only monitors the process mean, and monitoring process variability requires a different technique.

This study aims to differentiate between in-control and out-of-control states by comparing the distributions of pixels in different images.

The EWMA control chart examines each component of U_{kl} individually and identifies components that exceed the upper control limit (UCL). According to the findings of Megahed et al. [13], the EWMA statistic is defined as follows:

$$Z_i = (1 - \lambda)Z_{i-1} + \lambda\bar{X}_i \quad i = 1, 2, \dots, n \quad (3)$$

Where Z_0 is equal to μ_0 , \bar{X}_i is the i^{th} sample mean, Z_i and Z_{i-1} are the values of EWMA statistics at time t and $t-1$ respectively. Also, λ is the smoothing parameter that controls the exponential weight of the past observations. The mentioned control chart is used in two separate steps. It is first used to estimate different parameters in phase I, which generates in-control images. Then it is used to detect the occurrence of a process shift to present a good estimate of all fundamental metrics, including accuracy of the change-point, MSE value, and so on (phase II). It is noteworthy that components of a compressed image, presented as the 1D vector for the image, are used in process monitoring.

4. Numerical example

In this section, a numerical example has been solved to investigate the effect of image processing on process improvement in industries and the results have been analyzed in the tile manufacturing industry. The process begins with a 3264×2448-pixel image captured from a tile. As part of the initial preprocessing, we remove unwanted elements such as the table and fixture from the image.

Subsequently, the RGB image is converted to grayscale and resized to 256×256 pixels. In this grayscale image, the pixel intensity varies from 0 (representing black) to 255 (representing white). To normalize the pixel intensities, each number in the intensity matrix I is divided by 255, transforming them into values between 0 and 1. This results in the creation of an image with intensity values ranging from 0 to 1. Lastly, contrast enhancement is performed on the image.

1000 in-control images were created in this research by introducing noise to the original image. Various types of noise, including salt and pepper, Poisson, and Gaussian noise, were added as white noise to the original image to generate in-control phase I data. We considered a Gaussian distribution with $\mu = 0$ and $\sigma_0^2 = 0.01$ in our simulation study. The standard deviation of white noise is a quantity for detecting out-of-control conditions in the image intensity matrix.

By examining the in-control images produced when white noise is added to the base image, the upper control limit (UCL) for the EWMA control chart is determined based on an in-control average run length (ARL) of 200. Subsequently, images with various designated shifts in the pixel intensity matrix are created.

We need to determine the mean square error (MSE). MSE is calculated using the following equation to estimate the quality of compressed images. The criteria MSE is chosen to estimate the quality of the reduced image.

$$MSE(u, \hat{u}) = \frac{\sum_{i=1}^N \sum_{j=1}^M (u(i, j) - \hat{u}(i, j))^2}{NM} \quad (4)$$

Then, the performance of the proposed control chart was evaluated based on different intensity shifts. The nominal image after preprocessing was defined with a resolution of $N = M = 256$ and the number of components of the reduced image (U_{kl}) is specified as

$$n = m = \frac{N}{\sqrt{\rho}} = \frac{M}{\sqrt{\rho}} = \frac{256}{\sqrt{16}} = 64 \text{ based on the compression ratio } \rho = 16. \text{ When the image is}$$

compressed, each row of the image intensity matrix contains 64 elements. Each element represents a 4×4 block of pixels in the original image.

The performance of the proposed method to assess the effectiveness of image processing on process improvement based on different values of λ in the EWMA control chart and 20 images are summarized in Tables 2, 3, and 4.

Table 2. MSE values based on $\lambda = 0.01$

Image number	Intensity shift (Δ)									
	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1
1	64	68	73	72	96	111	115	123	148	151
2	72	86	88	93	103	104	119	137	142	144
3	68	72	71	85	89	88	103	117	124	135
4	52	57	62	66	73	78	93	95	101	105
5	96	99	103	128	142	146	187	193	214	232
6	85	89	99	108	113	124	123	145	187	193
7	54	59	66	74	79	95	99	132	174	168
8	89	93	95	121	118	135	138	149	157	175
9	43	54	73	88	110	115	112	118	124	134
10	82	88	93	97	101	115	127	138	142	153
11	123	143	141	159	178	183	185	196	207	234
12	110	115	124	136	142	140	153	176	182	193
13	68	65	79	85	89	93	115	127	136	142
14	76	77	83	92	95	94	106	124	128	145
15	96	105	113	127	139	145	143	169	182	199
16	85	86	95	101	113	124	138	174	186	195
17	65	69	75	96	94	103	117	128	138	147
18	34	38	46	79	85	86	92	115	121	125
19	69	74	89	103	115	128	139	145	155	169
20	75	96	98	127	126	138	149	168	196	206

Table 3. MSE values based on $\lambda = 0.05$

Image number	Intensity shift (Δ)									
	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1
1	68	73	75	77	96	112	119	126	147	156
2	72	86	85	91	100	102	112	139	143	143
3	68	72	70	85	89	88	103	117	126	135
4	51	57	61	66	73	78	93	95	107	105
5	99	102	103	128	142	146	187	193	215	232
6	82	89	99	108	113	124	123	145	189	193
7	52	59	66	74	79	95	99	132	181	168
8	83	93	95	125	118	135	148	149	166	185
9	37	67	69	93	113	115	125	128	129	144
10	82	88	93	97	101	115	127	138	142	153

Image number	Intensity shift (Δ)									
	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1
11	128	147	149	169	188	193	188	199	217	245
12	110	115	124	136	142	140	153	176	182	193
13	68	65	79	85	89	93	115	127	136	142
14	76	77	83	92	95	94	106	124	128	145
15	96	103	112	125	138	142	141	168	175	196
16	79	86	98	101	113	124	138	174	186	193
17	63	73	79	96	94	103	117	128	138	145
18	38	43	49	82	88	92	98	125	146	173
19	72	69	89	103	115	128	139	145	155	168
20	74	95	98	127	126	138	149	168	196	204

Table 4. MSE values based on $\lambda = 0.10$

Image number	Intensity shift (Δ)									
	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1
1	63	65	73	72	96	113	117	125	149	156
2	71	87	89	94	103	104	119	137	142	146
3	67	76	73	86	89	88	103	117	124	134
4	53	54	64	67	73	78	93	95	101	103
5	99	101	109	129	142	146	187	193	214	237
6	86	88	101	108	113	124	123	145	187	192
7	58	61	67	76	79	95	99	132	174	162
8	91	93	98	121	118	135	148	149	167	185
9	56	58	75	95	113	125	127	138	141	134
10	83	89	95	97	101	115	127	138	142	155
11	129	147	168	179	196	198	197	211	225	263
12	111	117	124	136	142	140	153	176	182	196
13	71	66	75	85	89	93	115	127	136	143
14	78	79	85	92	95	94	106	124	128	142
15	103	109	119	129	135	143	144	169	172	193
16	87	87	96	101	113	124	138	174	186	193
17	66	71	76	96	94	103	117	128	138	145
18	42	48	52	83	95	96	102	129	153	176
19	72	76	102	103	115	128	139	145	155	168
20	75	98	102	127	128	141	151	168	197	204

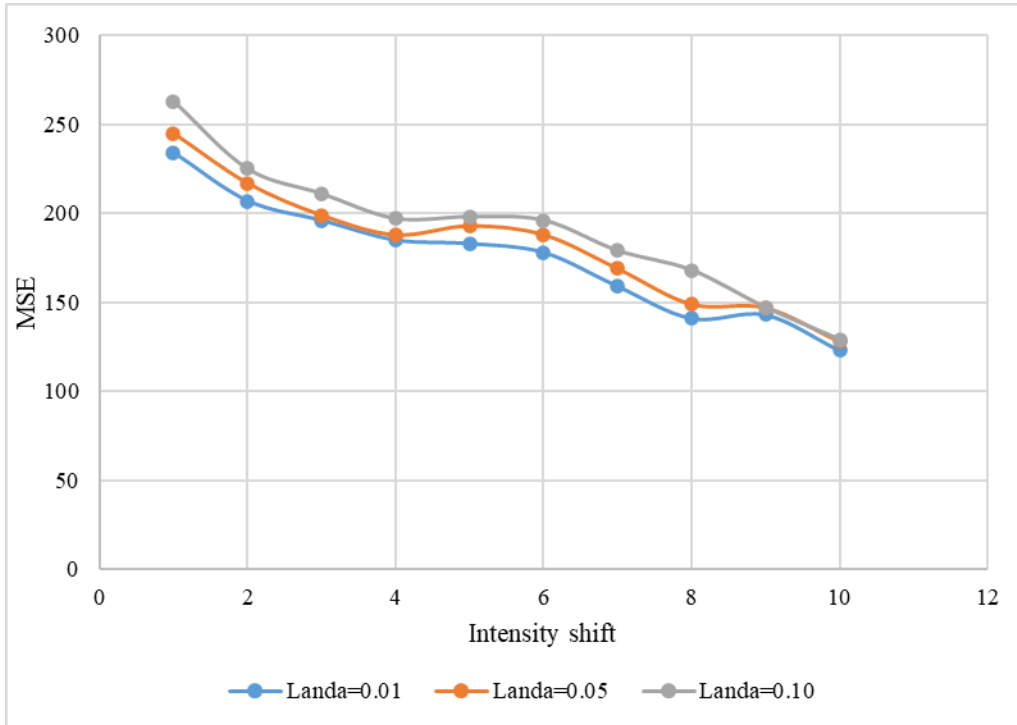


Figure 1. Comparison the best value of MSE based on different λ

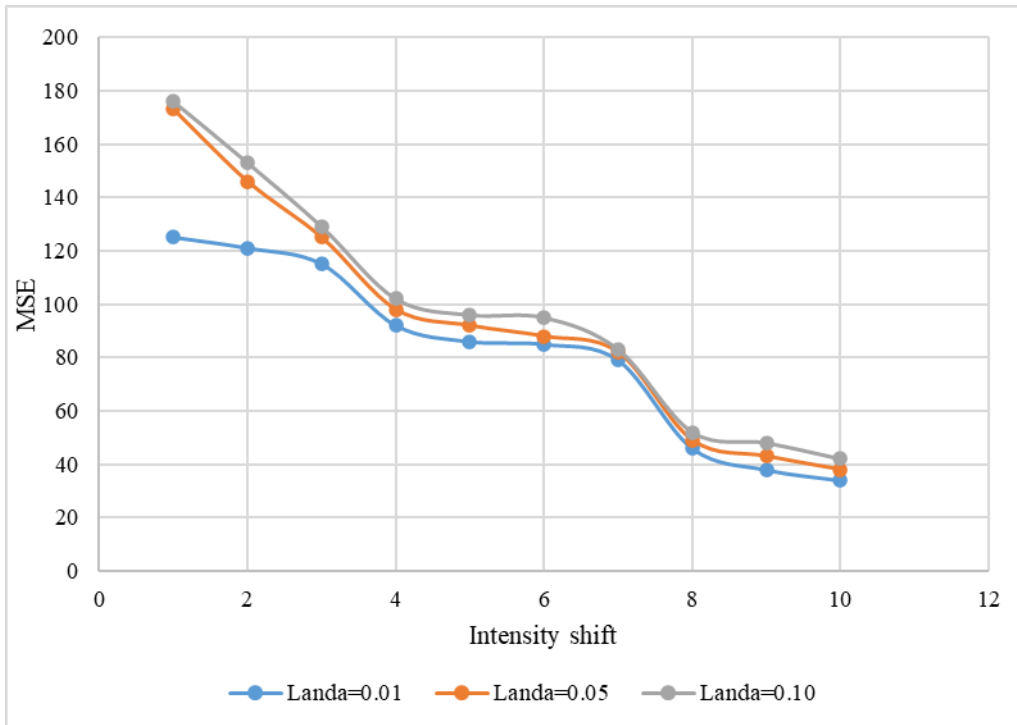


Figure 2. Comparison the worst value of MSE based on different λ

The solution results of the proposed model have been calculated in Tables 2, 3, and 4 for different values λ using MATLAB software. The MSE values that were obtained represent the image

processing performance in terms of image compression. Various values of intensity shift ranging from +1 to +10 were considered, and the MSE index was calculated for different images.

The evaluation of the results indicates that for all the images, increasing the intensity shift value from +1 to +10 leads to a decrease in error. In other words, as the intensity shift in the images increases, the error detection power in the proposed model improves, resulting in a decrease in the error rate, which is logical. Therefore, it can be concluded that the image processing method shows better performance for large-intensity shift values compared to small-intensity shift values.

To compare the performance of the proposed approach, a comparative study was conducted based on the best and worst MSE values for all three λ values. In the comparative study, the worst case is related to image number 11, and the best case is related to image number 18, with these images having the largest and smallest MSE values, respectively. The results of this comparative study are shown in Figures 1 and 2.

According to the obtained results, it can be understood that increasing the intensity shift from +1 to +10 leads to a decrease in the MSE, showing a downward trend. Additionally, the results indicate that for both the worst and best cases, the MSE for λ equal to 0.01 demonstrates better performance, suggesting that the detection power of the proposed approach is better for this value. Therefore, utilizing image processing based on small values of λ shows better performance in improving processes in the manufacturing industry.

5. Conclusions

The study presents a novel image-processing method for enhancing processes in the manufacturing sector. The system utilizes an exponentially weighted moving average (EWMA) control chart to identify any alterations in the monitored image data. The method is tested with a real case study in the tile manufacturing industry and various numerical examples to evaluate its effectiveness. The performance evaluation indicates that the image processing method performs better for larger intensity shift values than for smaller ones. The results also reveal that increasing the intensity shift from +1 to +10 leads to a decrease in the MSE, indicating an overall downward trend. Moreover, the MSE for a λ value of 0.01 shows better performance in both the worst and best cases, suggesting that this approach has superior detection power. Therefore, utilizing image processing based on smaller λ values demonstrate improved performance in process improvement for the manufacturing industry.

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