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Application of Markov chains in manufacturing systems: A review

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Abstract

Manufacturing is an essential aspect to the global economy and prosperity. Many Manufacturing systems operate in an uncertain environment which affects the system performance. Production planning is very key in improving the overall manufacturing system performance. Systems that apply production planning approaches not considering uncertainties yield inferior planning decisions as compared to those that explicitly account for the uncertainty. Markov chains can be used to capture the transition probabilities as changes occur. Some existing literature on application of Markov chains in manufacturing systems has been reviewed. The objective is to give the reader beginning points about uncertainty modelling in manufacturing systems using Markov chains.

1. Introduction

Manufacturing is described as the procedure of using raw materials, components or sub-components to produce finished products that meet the customers' requirements [1]. Characterization of manufacturing systems, like many other systems, can be dynamic or static, stationary (time-invariant)[2] or non-stationary (time-varying), linear or non-linear, discrete-state (time) or continuous-state (time), event-driven or time-driven, and stochastic or deterministic [3]

Manufacturing companies are facing a growing and rapid change where trends like globalization, customer orientation and increasing market dynamics have led to a move in both managerial and manufacturing principles which calls for more flexibility, fast and effectiveness [4].

Product demand uncertainty is one of the challenges faced by manufacturing companies [5] and influences the performance of the manufacturing system and the final decision on utilizing the manufacturing system [6].

The criteria of performance like manufacturing lead times, inventory costs, customer satisfaction, machine utilization, meeting due dates, and quality of products all dependent on how efficiently the jobs are scheduled in the system [7]. Therefore it becomes increasingly important to develop effective production planning approaches that help in achieving the desired objectives.

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Production planning has an important role in the manufacturing system. The more variety of products, increased number of orders, increased number and size of workshops and expansion of factories have all made production planning more complicated, making the traditional methods of optimization unable to solve them [4]

Production planning in manufacturing systems is affected by a number of uncertainties which need to be considered in order to generate better planning decisions. [8]

Markov chain is a powerful mathematical tool that is extensively used to capture the stochastic process of systems transitioning among different states [9].

When manufacturing systems reveal some random behavior (breakdowns, random time to process a part), markov chains can be used for modeling and performance evaluation [10]. Companies' model manufacturing processes for many reasons, including predicting cost, predicting resource and material demand and running optimization studies. Basing future business simulations on these markov chains can give a more reliable representation of the business which reduces the risk of modelling inaccuracies and can help to predict future outcomes and run optimization more accurately [1].

Due to significant importance of Markov chain models and their applications in manufacturing, additionally because the substantial amount of theoretical and practical results, it's of interest to supply a summary of their applications, and discuss future directions. To realize this, an instantaneous opening move is to classify the related literature and describe or review the main results. Such a piece can help readers to quickly picturize the realm, find interesting topics, and do further in-depth and detailed studies in specific areas. Particularly it can benefit those that are new the sector the most. Therefore, the main purpose of this paper is to produce a review on application of Markov chains in manufacturing systems.

To gain a better understanding of the application of Markov chains in manufacturing systems and to provide a basis for future research, a broad review of some existing research on the topic has been presented.

2. Basic concepts

The concepts and theory applied in this study are presented in the section below. This study was centered on the theory of Markov Chains focusing on their application in manufacturing systems. A Markov chain may be a special sort of model. Manufacturing systems, the concepts of stochastic process, Markov chain, types of Markov chains, Markov chain model states, transition probability matrix, properties of Markov chains, classification of states and application areas are presented in sub sections as outlined below.

2.1 Manufacturing systems

A manufacturing system may be a network of interacting parts. Managing the network of interacting parts is as important as managing individual parts, if not so more. In manufacturing systems research, a lot of interesting fields come to mind, such as design, analysis, modeling, optimization and control [11].

Manufacturing systems contain a number of several system factors among which exists work environment, physical structure, performance measurements, work organization, market & strategy, and manufacturing development process.[12]

Most of the manufacturing companies are large, complex systems characterized by a number of decision subsystems, like finance, personnel, marketing, operations and operates in an uncertain environment.[13]

A manufacturing system is an objective oriented network of processes through which entities flow with an objective of improving throughput or flow time.

It also contains processes that are not only physical, but can include support of direct manufacturing (e.g., order entry, maintenance). Due to variability in manufacturing systems, values of performance measures fluctuate, resulting in complexity. Therefore, models are required to imitate behavior of manufacturing systems. Together with variability, the evolution of manufacturing systems leads to a need for predicting behavior of the manufacturing systems [11].

2.2 Stochastic process

A stochastic process may be a mathematical model that evolves over time in probabilistic manner [14]. A stochastic process is a random process [10], that is, a change in the state of some system over time whose course depends on chance and for which the probability of a particular course is defined. Essentially it is a family of random variables, X (t): t Є T defined on a given probability space, indexed by the time variable t, where t varies over an index set T [15].
A stochastic process may be continuous or discrete. A stochastic process is claimed to be a discrete time process if set T is finite or countable. That is, if T= (0, 1, 2, 3 ,4…….. , n) resulting in the time process X(0), X(1), X(2), X(3), X(4), …….., X(n) , recorded at time 0,1,2,3,4……,n respectively [16]. On the other hand stochastic processes X (t): t Є T is considered a continuous time process if T is not finite or countable. That is, if T= [0, ∞) or T= [0, k] for some value k.
A state space S is the set of states that a stochastic process can be in. The states can be finite or countable hence the state space S is discrete, that is S=1, 2, 3…, N. Otherwise the space S is continuous [17].

2.3 Markov chain

Markov chain, named after a Russian mathematician Andrey Markov in 1907, is a powerful mathematical tool that is used widely to capture the stochastic process of systems transitioning among different states [9]. Markov chains were recognized rapidly for their significant power of representation and their possibility of modeling a wide range of real life problems in addition to the quality of performance indices they give [10]. When manufacturing systems reveal some random behavior, Markov chains can be used to carry out performance evaluation and modeling [18].
A Markov chain, special type of stochastic process (with a Markov property [19]), is a discrete-time stochastic model defined on a space of states, equipped with transition probabilities from one state to another at the next time stage [20].
Markov Chains have revealed their strength at modeling stochastic transitions, from uncovering sequential patterns to directly modeling decision processes [21]. These have got a special property that probabilities involving how the process will evolve in the future depend only on the present state of the process, and so are independent of events in the past [22].
A Markov process is a stochastic process that satisfies the Markovian property (says that the conditional probability of any future "event," given any past "event" and the present state Xt-i, is independent of the past event and depends only upon the present state[17], [15]). It is a sequence of random variables $X_1, X_2, X_3, \dots . X_n$ with the Markovian property, namely that, given the present state, the future and past state is independent. Formally[23],

$$P_r\left(X_{n+r} = \frac{x}{X_1}, = x_1, X_2 = x_2, \dots . X_n = x_n\right) = P_r\left(X_{n+r} = \frac{x}{X_n}, = x_n\right), \quad (1)$$

if both conditional probabilities are defined, i.e. if $P_r(X_1 = x_1, \dots . X_n = x_n) > 0$ the possible values of X_n form a countable set S called the state space of the Chain [4].
Markov Chains often described by a sequence of directed graphs, where the edges of the graph n labeled by the probabilities of going from one state at time n to another state at time$(n + 1)$,

$$P_r\left(X_{n+r} = \frac{x}{X_n}, = x_n\right) \quad (2)$$

However, Markov Chains assumes time-homogenous scenarios[24], in which case the graph and matrix are independent of n and not presented as sequences [4].

2.3.1 Types of Markov chains

There are two differing types when approaching Markov chains which is, discrete-time Markov chains and continuous-time Markov chains. This means that there are scenarios where the changes happen at specific states and others where the changes are continuous [25].

Discrete-Time Markov Chains (DTMC)
These are Markov chains that are observed only at discrete points in time (e.g., the end of each day) rather than continuously. Each time it is observed, the Markov chain can be in any one of a number of states [26].

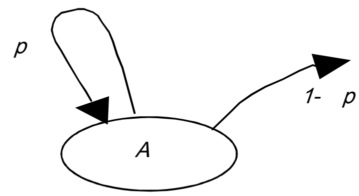

Fig. 1. Discrete-Time Markov Chains [27]

P {system stays in state A for N time units | as long as the system is currently in state A} = p^N
P {system stays in state A for N time units before exiting from state A} = $p^N(1-p)$

State changes are pre-ordained to occur only at the integer points 0, 1, 2, ..., n (that is at the time points t_0, t_1, t_2,......, t_n)[28] [29]
The sequence of random variables X_1, X_2, ... forms a Markov Chain if for all n (n = 1, 2, ...) and all possible values of the random variables, giving;

$$P\left\{\frac{X_n=j}{X_1=i_1 \_\_\_\_\_\_ X_{n-1}=i_{n-1}}\right\} = P\left\{\frac{X_n=j}{X_{n-1}=i_{n-1}}\right\} \quad (3)$$

Continuous-time Markov Chains (CTMC)
A continuous-time Markov chain changes at any time (State changes may occur anywhere in time) [26].
A Markov chain with continuous time is a stochastic process with Markov characteristics whose future state conditional probability, depends on present state which have no relation to past state of process [30].

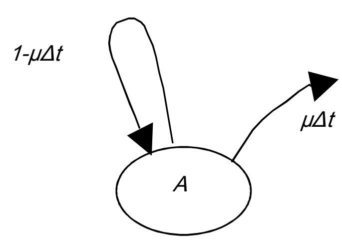

Fig. 2. Continuous -Time Markov Chains [27]

P{system in state A for time T | system currently in state A}

$$= (1-\mu\Delta t)^{\frac{T}{\Delta t}} \rightarrow e^{-\mu T} \qquad \Delta t \rightarrow 0 \quad (4)$$

2.3.2 Markov chains exploration

Markov chains model discrete-time processes and Markov processes models continuous-time processes. They mathematically model a process by showing how the method can move between different stages and therefore the probability of creating these transitions.
Markov's analysis can be represented diagrammatically as in figure 1 which shows a Markov chain model of a process with two stages A1 and A2, where the probability of making a transition from stage *i* to stage *j* is q_{ij} [1].

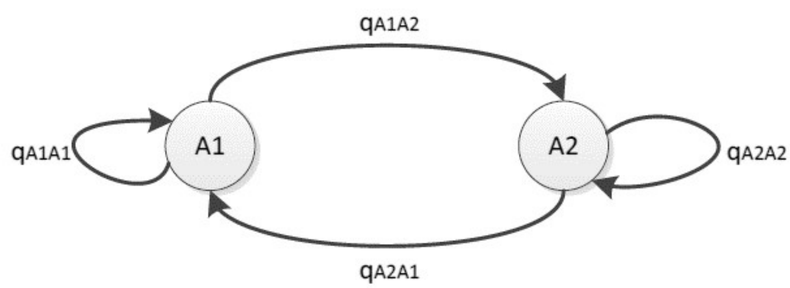

Fig. 3. Markov Chain Diagram [1]

2.4 Markov Chain Model States

The Markov chain model is a sequential process that consists of many steps. For those steps considered as Markov Chain states, they should respect all the following three conditions:

1. "State i communicates itself"
2. "If state i communicates with state j, then j communicates with state i."
3. "If state i communicates with state j, and j communicates with state k, then i communicates with state k."

According to [4], the probability of going from state i to state j in n time steps is given by: $\mathrm{P}_{ij}^{(n)} = P_r\left(X_n = \frac{j}{X_0}, = \mathrm{i}\right)$ and the single step transition is $P_{ij} = P_r\left(X_1 = \frac{j}{X_0}, = \mathrm{i}\right)$
For a time-homogenous Markov Chain, the probability is: $\mathrm{P}_{ij}^{(n)} = P_r\left(X_{n+k} = \frac{j}{X_k}, = \mathrm{i}\right)$ and $P_{ij} = P_r\left(X_{k+1} = \frac{j}{X_k}, = \mathrm{i}\right)$. A Markov Chain of order m, where m is finite, may be a process satisfying

$$P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \dots . X_1 = x_1\right)$$
$$= P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \dots . X_{n-m} = x_{n-m}\right) \; for \; n > m$$

In other words, the future state depends on the past m states. It is possible to construct a Chain Y_n from X_n which has the 'classical' Markov property by taking as state-space the ordered m tuples of x values, i.e. $Y_n = (X_n, X_{n-1}, \dots, X_{n-m+1})$ [4]

2.5 Transition Probability Matrix

Transition probabilities are conditional probabilities P (X t+1 = j/ X t =i} = P_{ij} arranged in the form of a n x n matrix called the transition probability matrix given by:

$$\begin{pmatrix} p11 & p12 \dots & p1n \\ p21 & p22 \dots & p2n \\ pn1 & pn2 \dots & pnn \end{pmatrix}$$ which can be denoted as P = P_{ij}

The transition matrix shows the probability of transitioning between the row stage to the column stage. To form a Markov chain model the transition probabilities are required and are calculated using the equation below which determines the probability of making a transition from stage i to stage j, which is represented by P_{ij}. Where m is the total number of transitions and n_{ij} is the number of transitions from i to j [1].

$$p_{ij} = \frac{n_{ij}}{\sum_{k=1}^{m} n_{i,k}} \tag{5}$$

The Transition Probability Matrix has the following properties: [15]

1. $P_{ij} > 0$ for all i and j.
2. For all i and j, sum of the element in each row is equal to 1. The sum represents total probability of transition from state i to itself or the other state.
3. The diagonal element represents transition from one state to same state.

Markov Chain models are useful in studying the evolution of systems over repeated trials. The repeated trials are often successive time periods where the state of the system in any particular period cannot be determined with certainty. Rather, transition probabilities are used to describe the way during which the system makes transitions from one period to subsequent. It helps us to determine the probability of the system being in a particular state at a given period of time [31].

2.6 Properties of Markov chains

Periodicity

A state i has period k if any return to state i must occur in multiple of k time steps. Formally, the period of a state is defined as:

$$\mathrm{K} = \gcd\left\{n: P_r\left(X_n = i / X_0 = i\right) > 0\right\} \tag{6}$$

(where "gcd" is the greatest common division). Note that even though a state has period k, it may not be possible to succeed in the state in k steps. For example, suppose it is possible to return to the state in {6, 8, 10, 12….} time steps; k would be 2, even though 2 does not appear in this list.

If k = 1, then the state is claimed to be a periodic: returns to state i can occur at irregular times, in other words, a state i is aperiodic if there exists n such that for all $n^1 \geq 0$

$$P_r\left(X_n^1 = i / X_0 = i\right) > 0 \tag{7}$$

Otherwise$(k > 1)$, the state is said to be periodic with period k. a Markov Chain is aperiodic if every state is aperiodic. An irreducible Markov Chain only needs one aperiodic state to imply all states are aperiodic. Every state of a bi partite graph has an even period. [4], [15].

Recurrence

A state i is said to be transient if, given that the system start in state i, there is a non-zero probability that the system will never return to i formally, but the random variable Ti be the first return time to state i (the "hitting time"): [4]

$$T_i = inf\left\{n \geq 1: X_n = i / X_0 = i\right\} \tag{8}$$

The number $f_{ii}^{(n)} = P_r(T_i = n)$ is the probability that state is returned to for the first time after n steps. Therefore, state i is transient if

$$P_r(T_i < \propto) = \sum_{n=1}^{\infty} f_{ii}^{(n)} < 1 \tag{9}$$

State i is recurrent if it is not transient. Recurrent states are guaranteed to have a finite hitting time [15].

Ergodicity

A state i is said to be ergodic if it is periodic and positive recurrent. In other words, a state has a period of 1 and it has finite mean recurrence time. If all states in an irreducible Markov chain are ergodic, then the chain is claimed to be ergodic. It can be shown that a finite state irreducible Markov chain is ergodic if it's a periodic state.

A model has the ergodic property if there's a finite number such that any state can be reached from any other state in exactly N steps. In case of a fully connected transition matrix where all transitions have a non-zero probability, this condition is fulfilled with N = 1. That is a Markov chain is ergodic if there exists some finite k such that;

$$P\left\{X(t+k)=j \big/ X(t)=i\right\} > 0 \text{ for all i and j [15]}$$

A model with more than one state and just one out transition per state cannot be ergodic.

Reducibility

A state j is claimed to be accessible from a state a system started in state i has a non-zero probability of transitioning into state j at some point. Formally, state accessible from state i if there exists an integer $n_{ij} \geq 0$ such that

$$P_r(X_n = j/X_0 = i) = p_{ij}^{n_{ij}} > 0 \tag{10}$$

This integer is allowed to vary for every pair of states, hence the subscripts in nij. Allowing *n* to be zero means that every state is defined to be accessible from itself. A state i is said to communicate with state j (written $i \leftrightarrow j$) if both $i \rightarrow j$ and $j \rightarrow i$. A set of states *C* may be a communicating class if every pair of states in *C* communicates with each other, and no state in *C* communicates with any state not in *C*. It may be shown that communication in this sense is an equivalence relation and thus that communicating classes are the equivalence classes of this relation. A communicating class is closed if the probability of leaving the category is zero, namely that if *i* is in C but j isn't, then *j* isn't accessible from *i*.

A state i is claimed to be essential or final if for all j such that $i \rightarrow j$ it's also true that $j \rightarrow i$. A state *i* is inessential if it's not essential. A Markov chain is claimed to be irreducible if its state space may be a single communicating class; in other words, if it's possible to get to any state from any state [15].

2.7 Classification of states of a Markov chain

Recurrent States

A state is claimed to be a **recurrent state** if, upon entering this state, the method definitely will return to the present state again. Therefore, a state is recurrent if and as long as it's not transient. Since a recurrent state definitely will be revisited after each visit, it will be visited infinitely often if the process continues forever [32].

If the method enters a particular state then stays during this state at the subsequent step, this is often considered a return to the present state. Hence, the following kind of state is a special type of recurrent state [26].

Transient States

A state is claimed to be a **transient state** if, upon entering this state, the method may never return to the present state again. Therefore, state i is transient if and as long as there exists a state j (j≠ i) that's accessible from state i but not the other way around, that is, state i is not accessible from state j [33].

Thus, if state i is transient and the process visits this state, there is a positive probability (perhaps even a probability of 1) that the process will later move to state j and so will never return to state i. Consequently, a transient state will be visited only a finite number of times[26].

When starting in state i, another possibility is that the process definitely will return to this state [34].
The Markov process is transient if the state can only be visited a finite number of times otherwise, the state is recurrent[11].

Absorbing states
In an absorbing Markov chain model, the Markov chain may include circles and it theoretically allows an infinite number of circulations among certain process states [35]
A state is claimed to be an **absorbing state** if, upon entering this state, the process never will leave this state again. Therefore, state i is an absorbing state if and only if $P_{ii} = 1$ [33]
A Markov chain with one or more absorbing states is understood as absorbing Markov chain. An absorbing state is, because the name implies, one that endures. In other words, when a work-part reaches such a state, it never leaves the state [36].

2.8 Areas of application of Markov chains

Markov chains are used in a variety of situations since they can be considered to model many real-world processes. These fields include, to mention but a few, quality management [37], system performance (reliability & availability)[38], electronics [35], condition monitoring [39], physics, chemistry, computer science, queuing theory, economics, games, and sports [23].

3. Markov models in manufacturing

In an effort to gain a better understanding of the markov chains and its application in manufacturing, and to provide a basis for future research, a broad review of some existing research on the subject has been presented.
Table 1 gives a summary of citations on Markov models in manufacturing. A complete of 42 citations on Markov models in manufacturing were reviewed. The majority of the citations were found in journals (78.571%), proceedings, conferences and others (11.905%), books (4.762%) and published PhD Thesis (4.762%).

Table 1: Summary of citations on Markov models in manufacturing

Source	Number of citations	% total
Journal of Industrial Engineering	1	2.381
Procedia Manufacturing	2	4.762
International Journal of computer science issues	1	2.381
Conference proceedings	5	11.905
Thesis	2	4.762
Journal of Mathematics and Statistics	1	2.381
Book	2	4.762
International journal of industrial engineering and operational research (IJIEOR)	1	2.381
UPB Scientific Bulletin, Series D: Mechanical Engineering	2	4.762
International Journal of Energy Research	2	4.762
Periodica Polytechnica Social and Management Sciences	1	2.381
Nuclear Engineering and Design	1	2.381
Journal of Advanced Mechanical Design, Systems and Manufacturing	1	2.381
Journal of the Operational Research Society	1	2.381
International Journal of Engineering Research & Technology	1	2.381
Advances in Science and Technology Research Journal	1	2.381
International Journal of Production Economics	2	4.762
Journal of Cleaner Production	2	4.762
International Journal of Current Research	2	4.762
Journal of Banking Financial	1	2.381

Computers and Chemical Engineering	1	2.381
Computers & Industrial Engineering	2	4.762
Quality Engineering	1	2.381
Manufacturing and Service Operations Management	1	2.381
Journal of Industrial Mathematics	1	2.381
Applied Sciences (Switzerland)	1	2.381
IJISET-International Journal of Innovative Science, Engineering & Technology	1	2.381
Acta Mathematica Scientia	1	2.381
Annals of the Academy of Romanian Scientists Series on Engineering Sciences	1	2.381
Total	**42**	**100**

Table 2 provides a summary of the classification scheme of the literature addressed in this study about the Markov models in manufacturing, giving the research topic, nature of uncertainty, research approach and conclusions drawn.

Table 2: Classification scheme of literature on Markov models in manufacturing

References	Research topic	Uncertainty	Approach detail	Conclusion
Leigh et al., 2017 [1]	Modelling manufacturing processes	Human interaction & variable products	Radio Frequency Identification (RFID)	Created a Markov chain model used to predict future product paths for use in discrete event simulation
Tochukwu et al., 2015 [4]	Agent Based Markov Chain for Job Shop Scheduling and Control	Dynamic market changes	Scheduling algorithms	Developed an agent based model where all information of the dynamics of the model was formulated as a Markov chain
Kiassat et al., 2013 [19]	Effects of operator learning on production output	Operator learning	Proportional hazards model	Developed a Markov chain approach to forecast production output of a human-machine system, considering HR factors and operator learning.
Gingu & Zapciu, 2017 [10]	Synchronizing the manufacturing production rate with real market demand	Market demand	Markov chains and decomposition method, C++	Offered a solution, by avoiding intermediary stocks at the same time, and a predictable market demand of these products (balancing between demand and production)
Ye et al., 2019 [9]	Modeling for reliability optimization of system design & maintenance based on Markov chain theory	System failures and repairs	Continuous- time Markov chain	Proposed a non-convex MINLP model
Chatys, 2020 [2]	Application of the Markov Chain Theory in Estimating the Strength of Fiber-Layered Composite	Static strength and fatigue life	Vacuum bag method (mathematical model)	MM can be used for “predicting” the S-N curve, taking into account the

	Structures with Regard to Manufacturing Aspects			maximum volume share of reinforcement in the composite and manufacturing technology
Sastri et al., 2001 [34]	Markov chain approach to failure cost Estimation in batch manufacturing	Failure cost estimation (repair/rework)	Markov chain approach,	Showed how a markov chain model is used to estimate a fore mentioned activity based failure costs
Santhi, 2019 [16]	Markov decision process in supply chain management	Inventory levels	Markov decision process	Determined the service rates to be employed as a function of the number of customers in the queue and the amount of inventory on hand minimizing the long-run expected cost rate.
Mubiru, 2013 [5]	An EOQ Model For Multi-Item Inventory With Stochastic Demand	Demand	Markov decision process	Demonstrates the existence of an optimal state dependent EOQ, produces optimal ordering policies and the corresponding total inventory costs for items.
Boteanu & Zapciu, 2017 [18]	Modeling and simulation of manufacturing flows for optimizing the number of work pieces on buffers from manufacturing systems	Failures, demand modifications, breakdown	Markov chains, Decomposition method, discrete event simulation	Dynamic adaptation of the production rate by optimizing the buffers according to the effective demand or estimated demand of the market.
Janicijevic et al., 2014 [37]	Using a markov chain for product quality Improvement simulation	Customer requirements	Simulation	Modelled the stochastic processes of a system of quality management and selection of the optimum set of FIPQ.
Sharma & Vishwakarma, 2014 [38]	Application of Markov Process in Performance Analysis of Feeding System of Sugar Industry	Systeme performance (failures)	Markov modelling	The system can be analyzed easily by concerning the process as Markov process and it helps the system design analyst to select the most appropriate structural components.

Pillai & Chandrasekharan, 2008 [36]	An absorbing Markov chain model for production systems with rework and scrapping	Scrapping and reworking	Probabilistic model	Identifies production system parameters under scrapping and reworking, and accurately estimates the quantity of raw materials required.
Afrinaldi, 2020 [23]	Exploring product lifecycle using Markov chain	Behavior of the product	Markov chain	Number of trips & duration of stay of a product in a lifecycle stage, no. of products visiting a specific lifecycle stage, probability of a product being discarded, and the expected total environmental impact of the product are predicted
Sobaszek et al., 2020 [24]	Predictive Scheduling with Markov Chains and ARIMA Models	Machine failure	Markov process	Inclusion of machine failure in the production schedule results in the extension of the performance indicators, mean flow time, mean job completion time, and the central criterion describing the performance of the production system
Strachan et al., 2009 [39]	A Hidden Markov Model for Condition Monitoring of a manufacturing drilling process.	Tool wear and impending failure	Algorithm; hidden Markov model	presented an algorithm for the condition monitoring of a manufacturing drilling process that will be able to detect tool wear and impending failure
Jónás et al., 2014 [35]	Application of Markov Chains for Modeling and Managing Industrial Electronic Repair Processes	Repairs	Absorbing markov chain	Modeling repair, manufacturing & business processes as acyclic absorbing Markov chains can ground many process management activities, enable managers to determine the probability distribution of lead time of any repairing process.

Beijsens & Rooda, 2005 [11]	Markov based modeling of manufacturing systems dynamics	Manufacturing system properties	Markov theory	Control a discrete manufacturing system with a continuous controller. And the continuous model validated with a discrete-event model
Karim & Nakade, 2020 [25]	A Markovian production-inventory system with consideration of random quality disruption	Product quality disruption	Stochastic model	Under the situation of production time constraint, the integration of safety stock in an interruption prone production–inventory system, assists in improving the average cost function.
Abedi et al., 2009 [30]	Using Markov Chain to Analyze Production Lines Systems with Layout Constraints	Layout constraints	Hybrid model (Markov chain in queue theory)	Developed a queuing model by analyzing a real queuing system with layout limitations in specific conditions and applying Markov chain concepts
Akhlaghi & Rostamy-Malkhalifeh, 2019 [40]	A linear programming DEA model for selecting a single efficient unit	-	Linear programming	Proposed a new LP model for finding the most BCC-efficient DMU where the decision maker is able to find most BCC-efficient DMU by solving only one LP
Lotfi, Mardani, et al., 2021[41]	Robust bi-level programming for renewable energy location	Robust stochastic	BLP	Robust modeling addressed to cope with different scenarios and disruption in demand
Lotfi, Kargar, et al., 2021[42]	Resilience and sustainable supply chain network design by considering renewable energy	Robust stochastic	GAMS and fix-and-opt	Suggested a novel SCND that wants to pay more attention to resiliency and sustainability by considering RE
Present study	Application of Markov chains in manufacturing systems	Stochastic	Markov chains	As a basis for decision making, Markov Chain prediction method is no exception and a combination of results from using Markov Chain to predict with other

		factors can be more useful.

From the reviewed literature, there a number of uncertainties that affect the performance in manufacturing companies. From table 2 it is seen that system failure and repairs (45%) is the most researched nature of uncertainty affecting manufacturing, then market /customer requirements or demand (25%), inventory levels (5%), product quality (5%), and others (20%) It is also seen that both Discrete-Time Markov Chains (DTMC) and Continuous-time Markov Chains (CTMC) approaches are used although Discrete-Time Markov Chains was used more.

4. Conclusion

This paper has presented an extensive literature survey about the application of Markov chains in manufacturing systems. Markov chain is an established concept in operations research and probability theory and it has been applied to many areas in manufacturing including quality management, system performance (reliability & availability), supply chain, electronics, condition monitoring, queuing theory, economics, to mention but a few.
As a basis for decision making, Markov Chain prediction method is no exception and a combination of results from using Markov Chain to predict with other factors can be more useful.
More research should be done on development of models in the context of Continuous Time Markov Chains (CTMC) [5]. Models should further be developed to be applied for products having components and modules, the logistics operation behind the transition needs to be modeled so that the accuracy of the model is improved and, the economic aspects should be included in the model, to aid policymakers in making a comprehensive decision[23] .

5. References

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